Final Report

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PI: András Stipsicz

1 Introduction

In the four and a half year of the project we covered a rather substancial part of the subject of topological questions of manifolds. With the aid of the OTKA support more than 100 publications have been produced, which mostly appeared in rather prestigious international journals. The participants gave numerous lectures about their results, both within Hungary and abroad. The invitation of many scientists (who gave lectures at the Rényi Institute, mostly in the *Algebraic Geometry and Differential Topology* seminar) helped the participants of the project to deepen their existing international connections and collaborations, and also to start new cooperations.

The work detailed below not only prvoded solutions to problems, but also led to further studies, sometimes in the form of related results, sometimes in the form of conjectures.

In the following we will give details of the projects led by members of the research group participating in this project.

2 Detailed report

2.1 András Stipsicz

We continued our study of Heegaard Floer homologies and used different approaches to understand these important invariant. In a joint project with Ozsváth and Szabó we found further connections with lattice homology: we proved the existence of a spectral sequence from lattice homology to Heegaard Floer homology, and extended the knot theory package (which is the filtered chain homotopy type of a filtered chain complex) to the lattice context. We hope to continue this study for links, and ultimately we aim to prove the equivalence of the two theories. This approach resulted in the papers [1, 2]

We have also understood how to lift combinatorial Heegaard Floer homology from $\mathbb{Z}_2$ coefficients to $\mathbb{Z}$ coefficients. We showed that there is (up to a simple equivalence, called gauge equivalence) a unique sign assignment in a nice diagram, which satisfies the basic properties it needs to fulfill in order to produce a boundary operator with square zero. The results are given in [3].

We concluded our study of rational homology disk smoothings of isolated surface singularities. Relying on our earlier results, we showed that such a singularity must be weighted homogeneous, and in this class a complete classification was earlier obtained in a joint project with M. Bhupal.

New results in higher dimensional contact topology have been attained. In particular, we showed that spheres of dimension $8k-1$ with $k \geq 2$ always admit almost contact structures which do not contain Stein fillable contact structures. These results are subject of further researach, which we carry on in a joint project with D. Crowley and J. Bowden, and already published part of it in [4, 5].

2.2 Tamás Szamuely

We focused on algebraic groups and their homogeneous spaces, both from the geometric and the arithmetic point of view. Algebraic groups are the group objects in algebraic geometry.

In joint work with M. Brion, we proved that the prime-to-p algebraic fundamental group of an arbitrary connected algebraic group over an algebraically closed field of characteristic p (including $p = 0$) is always commutative, and moreover every étale Galois cover has the structure of a central isogeny. These results are parallel to classical topological facts and were previously only known in the special cases of linear algebraic groups and abelian varieties. We have also obtained a generalization to homogeneous spaces with connected stabilizers, and found a sharp bound for the topological rank of the prime-to-p fundamental group of these spaces.

Together with D. Harari, we studied local-global questions for Galois cohomology over the function field of a curve defined over a p-adic field. The novelty of this situation is that such fields have cohomological dimension 3, whereas in classical arithmetic situations the cohomological dimension is 2. We defined Tate-Shafarevich groups of a commutative group scheme via cohomology classes locally trivial at each completion of the base field coming from a closed point of the curve. In the case of a torus we established a perfect duality between the first Tate-Shafarevich group of the torus and the second Tate-Shafarevich group of the dual torus. As an application, we analyzed the failure of the local-global principle for rational points on principal homogeneous spaces under tori. That the local-global principle may indeed fail was shown by recent examples of Colliot-Thélène and Parimala. We showed that this failure is controlled by a certain subquotient of a third étale cohomology group. We also proved a generalization to principal homogeneous spaces under a large class of reductive group schemes in the case when the base curve has good reduction. This latter assumption is essential, as the local-global principle may fail for simply connected groups in the case where the base curve has bad reduction — a phenomenon that does not occur in arithmetic situations of cohomological dimension 2.

Together with D. Harari and C. Scheiderer, we also studied weak approximation of rational points on principal homogeneous spaces of tori over fields of the above type. We constructed a 9-term Poitou–Tate type exact sequence for tori over a field and a 12-term sequence for finite modules. Like in the number field case, part of the sequence could then be applied to analyze the defect of weak approximation for a torus. We also showed that the defect of weak approximation is controlled by a certain subgroup of the third unramified cohomology group of the torus, providing a link with recent work of Merkurjev and Blinstein.

2.3 András Juhász

Our first objective was to better understand the relationship between sutured Floer homology and taut foliations. Together with Altman and Friedl [6], for an oriented irreducible 3-manifold M with non-empty toroidal boundary, we described how sutured Floer homology (SFH) can be used to determine all fibred classes in $H^1(M)$. Furthermore, we showed that the SFH of a balanced sutured manifold (M, γ) detects, which classes in $H^1(M)$ admit a taut depth one foliation such that the only compact leaves are the components of $R(\gamma)$.

In order to study surfaces bounded by knots and links in D^4, we defined cobordism maps on knot Floer homology as a special case of cobordism maps on SFH. This was a major project that first involved settling the naturality of Heegaard Floer homology. This we did jointly with Dylan Thurston [9] via studying global bifurcations of generic 2-parameter families of gradient vector fields. In particular, we obtain a mapping class group action on Heegaard Floer homology.

Together with Kálmán and Rasmussen we found that, given a special alternating link diagram, from the SFH of the complement of the corresponding Seifert surface, we can obtain the "top" coefficients of the HOMFLY polynomial of the link. This proceeds via showing that the hypertree polytope defined by Kálmán coincides with the support of SFH. It also uses heavily the results obtained jointly with Friedl and Rasmussen [7] that characterizes the Euler characteristic of SFH as a relative version of Turaev torsion for sutured manifolds.

Since the proposal was written, the correspondence between SFH and bordered Floer homology has been verified by Lipshitz, Ozsváth, and Thurston, and the isomorphism between Heegaard Floer homology and monopole Floer homology has been proven independently by two groups, one being Kutluhan, Lee, and Taubes, the other Colin, Ghiggini, and Honda.

2.4 Szilárd Szabó

Fourier–Laplace–Nahm transform We identified Nahm transform of solutions of Hitchin's equations on the Riemann sphere with some singularities with Fourier–Laplace transform of the holonomic D-module obtained as the minimal extension of the connection on the open curve over the singularities. We drew the consequence that the transform respects the de Rham complex structure on the moduli spaces. In [11] we gave an interpretation of the same Nahm transform in terms of some birational transformations of the spectral sheaf of the associated Higgs bundle followed by a direct image morphism. This implies in particular that the transform respects the Dolbeault complex structure on the moduli spaces. Finally, (in a yet unpublished work) we showed that the transform respects the Dolbeault holomorphic symplectic structure on the moduli spaces too; it then follows that it is a hyperKähler isometry.

Fuchsian equations We answered a question of N. Katz from 1996 by proving that on the moduli space of logarithmic connections with fixed singular points and fixed exponents on a curve there exists a weight 1 real Hodge structure with the property that the submanifold of Fuchsian equations is everywhere tangent to the $(1, 0)$-part of the Hodge structure. We showed that rank 2 connections on the Riemann sphere with finitely many logarithmic singularities can be represented by Fuchsian equations with apparent singularities; this results in an explicit birational isomorphism of the moduli space of such connections (again with fixed singular points and fixed exponents) with an explicitly described open subset of the Hilbert scheme of points on the total space of a twisted cotangent bundle of the curve. In an ongoing work joint with Masa-Hiko Saito we extend this result to the case of higher order logarithmic connections on any compact curve.

Instantons on ALF 4-manifolds We studied the moduli space of instantons of energy 1 on the A_k asymptotically locally flat (ALF) 4-dimensional hyperKähler manifolds, also known as multi-Taub–NUT spaces; we showed that it has a description reminiscent of the compact case, namely the moduli space is a cobordism between the multi-Taub–NUT space itself and a disjoint union of cones (over the reducible solutions).

2.5 Vera Vértesi

With Etnyre and Ng we gave a complete classification of twist knots [12]. This result was the first classification of non Legendrian simple knot types. In a preprint with Etnyre we also classified transverse knots for several other families of knot types obtained by satellite construction.

In a joint [13] work with Baldwin and Vela-Vick — using only combinatorial techniques — we found an identification of the Legendrian invariants of Lisca-Ozsváth-Stipsicz-Szabó (LOSS) with the one defined by Ozsváth-Szabó-Thurston. This identification has been achieved by a convenient characterization of the invariant in both cases. This observation lead to an easier computation of knot Floer homology for braids.

In a preprint with Stipsicz we generalized lattice Floer homology for links, this definition gives hope for a proof of the equivalence of Heegaard Floer homology and lattice Floer homology by induction.

In a preprint with Ina Petkova we extended the idea of bordered Floer homology to knots and links in S^3: Using a specific Heegaard diagram for a tangle we constructed invariants of tangles in S^3, D^3 and $I \times S^2$. The special case of S^3 gives back a stabilized version of knot Floer homology. This new angle on knot

Floer homology allows one to do local computation. We demonstrated its power by giving a new proof for the skein exact sequence for knot Floer homology.

2.6 Róbert Szőke

In [14] we introduced the notion of smooth and analytic fields of Hilbert spaces, a weaker notion than bundles. We proved that a flat analytic field over a simply connected base corresponds to a Hermitian Hilbert bundle with a flat connection. By pushing forward a Hermitian holomorphic vector bundle along a non–proper map, we gave criteria for the direct image to be a smooth field of Hilbert spaces. Quantizing an analytic Riemannian manifold M, using the family of adapted complex structures (*acs*) on TM, we showed that for homogeneous M the direct image is an analytic field of Hilbert spaces, which is even flat for compact Lie groups. This means that in those cases quantization is unique. In another joint paper with L. Lempert, posted in the arxiv we show that for a Riemannian compact symmetric space of rank 1, the corresponding field is not even projectively flat (except for S^3). We also studied the *acs* of a left invariant metric given on a Lie group. This study would give some intuition for better understanding the curvature of the volume preserving diffeomorphism group of a compact manifold. This work is still in progress. A compact manifold (M, g) is entire if the *acs* is defined on TM. This implies that all the sectional curvatures of g are nonnegative. It is a conjecture that $\chi(M) \geq 0$. The work on this question is still in progress.

2.7 András Szűcs

The motivation of Poincaré for introducing homologies was purely geometric. The well-known moder definition is also motivated by the geometric picture. How far is geometry from algebra in this question? This was answered by Thom (in answering a question of Steenrod) when shoing that any homology class can be represented by a continuous image of a manifold. Is it possible to do the same with nicer maps, like immersions? What if we fix a class of multisingularities, can we represent each homology class with a map having multisingularity from the given list? In a joint project with M. Grant we answered these questions in the negative. In some sense it shows that the algebraic definition of homologies is infinitely apart from the geometric picture.

We also determined the cobordism group of maps with simplest possible singularities (with the aid of the group of homotopy spheres).

2.8 Mátyás Domokos

In most of this work we applied representation theory of classical or finite groups to the study of the coordinate ring of certain algebraic varieties.

The $n \times n$ degenerate real symmetric matrices form an interesting 2-codimensional real algebraic variety. Surprisingly little is known about the vanishing ideal of this variety. We determined generators of this ideal and the dimensions of its homogeneous components for $n = 3$. More generally, we investigated the varieties of real symmetric or Hermitian matrices with a bounded number of distinct eigenvalues. Applying a known fact on a combinatorially defined subspace arrangement we determined the minimal degree of a non-zero homogeneous component of this ideal.

Studies on transformation groups led us (with Endre Szabó) to the following question: is it true that when considering a Zariski closed orbit in the product of affine G-manifolds (where G is a reductive algebraic group) there is always a projection to a small (depending only on G) coordinate set, which is Zariski closed. Some positive partial results have been found in this direction, and these methods have been applied to the invariant theory of binary forms.

We also studied the algebra of semi-invariants of 3×3 matrix triples. It was known that this algebra is generated by twelve elements, the first eleven constitute a homogeneous system of parameters, and the last generator satisfies a quadratic relation over the parameter subalgebra. With a mixture of theory and computational methods, we succeeded in explicitly determining the above-mentioned quadratic relation. This relation also appears in the explicit construction of the Jacobian of a plane cubic. Finally, we recovered in a transparent way the characteristic-free description in terms of generators and relations of the ring of simultaneous conjugation invariants of pairs of 3×3 matrices, obtained previously by Nakamoto with hard and lengthy computations.

2.9 Tamás Terpai

The research revolved around the central theme of criteria for existence of maps with prescribed singularities in certain classes. In particular, necessary conditions were obtained for the existence of negative codimensional fold- and cusp-maps (joint work with Boldizsár Kalmár); these conditions are vanishings of certain Stiefel-Whitney characteristic classes of the source manifold and are hence easy to check algorithmically and in the case of codimension being -1 in most cases imply that the source manifold is null-cobordant. In the closely related problem of calculation of Thom polynomials and avoiding/obstruction ideals, we showed a partial degeneration of the Kazarian spectral sequence. This allows an easy deduction of the avoiding ideals of the Σ^r singularities and imbues some of their elements with geometric significance linked to the precise obstruction to removing these singularities by a bordism or cobordism, making the detection of unremovable singular strata more straightforward. We also calculate the obstruction ideal of the singularity $III_{2,2}$ in the case of maps over the real numbers without using the specificity of the case of complex maps. In a joint work with András Szűcs and Mark Grant the question of representing cohomology classes by images of maps with restricted singularities was investigated in the case of finitely many monosingularities (by a theorem of András Szűcs, finitely many multisingularities is not enough to represent k-dimensional classes for any $k \geq 2$), and a full answer was obtained when changing the manifold and the class by a cobordism is allowed.

2.10 László Fehér

In a joint paper with Rimányi and Domokos we determined the orbit-hierarchy for families of conics; we used equivariant cohomology. Explicit expressions have been also found for the invariant polynomials, and these results have been applied to compute new Thom polynomials and the multiplicities of determinant maps.

In a joint manuscript with Ákos Matszangosz we developed a method for giving lower bounds for the solution of certain enumerative problems over the reals using Pontryagin classes. To find applications it was necessary to develop methods of deciding whether certain real analytic sets represent integer cohomology classes, and also methods to calculate these classes. This is considerably more complicated than in the complex case.

With András Némethi we proved that in most cases maps between complex projective spaces are maximally singular, both in the holomorphic and the real smooth category.

In a joint work with András Szenes we calculated the integer valued Thom series of some real singularities. Partial results are available for all higher Morin singularities. However the existence (geometric meaning) of these series for higher Morin singularities is still unclear.

2.11 Alex Küronya

The main focus of our research was to understand the mysterious relationship between big line bundles on varieties and Newton–Okounkov bodies associated to them. One significant novelty was the introduction of functions on Newton–Okounkov bodies coming from geometric valuations, which lead to a surprising generalization of classical vanishing sequences on curves to arbitrary normal varieties and divisorial valuations.

As part of the above project with Sébastien Boucksom, Catriona Maclean, and Tomasz Szemberg, we studied the continuity of such functions, and gave an explicit formula for their integrals in terms of a certain restricted volume of the underlying line bundle. Since the functions mentioned above have a close relationship to Donaldson's work on test configurations, further developments are expected.

In a slightly different context, in a joint work with Marcin Dumnicki, Brian Harbourne, and Tomasz Szemberg, we linked the maxima of Okounkov functions to Seshadri-type constants associated to line bundles on surfaces, and proved a connection between their rationality and the rationality of classical Seshadri constants. This in turn then led to a connection between Nagata's conjecture and the structure of Okounkov functions.

Along a completely different line of thought, together with Daniel Greb we studied the vanishing of higher cohomology groups of sheaves on varieties, and established a very general Kodaira-type vanishing theorem for Du Bois q-ample pairs.

2.12 András Némethi

Together with Kerner, we found a counterexample for the 35-year-old Durfee conjecture which predicted an inequality between the Milnor fiber and geometric genus of a surface singularity. We corrected this inequality and proved the reformulated conjecture in the homogeneous case.

With M. Borodzik we proved that the Hodge theoretical spectrum of a local isolated plane curve singularity, and the mod-2-spectrum at infinity associated with an affine plane curve can be recovered purely from the topology of the involved algebraic links via their Tristram-Levine signatures.

We proved that for a normal surface singularity the Euler characteristic of the lattice cohomology coincides with the normalized Seiberg-Witten invariant of the link of the singularity. Moreover, this invariant can also be connected with a multivariable Poincare series associated with the resolution graph: the multivariable periodic constant of this series is exactly the above Seiberg-Witten invariant.

In a collaboration with J. Kollár we introduced the space of 'short holomorphic arcs' (the analog of formal arcspace) of a local analytic space germ, and in the case of surfaces determined the irreducible components (the analog of the famous Nash problem).

With Sigurdsson we proved that the geometric genus of a Newton non-degenerate or of a superisolated hypersurface singularity can be determined form the topology of the link via the Euler characteristic of the 'path lattice cohomology'.

2.13 Gábor Etesi

With Szilárd Szabó (by the aid of twistor theory and PDE techniques) we explicitly constructed the moduli space of SU(2) "admissible" instantons with unit energy and trivial holonomy at infinity over the multi-Taub–NUT space [15]. We went on and proved an energy identity for "admissible" instantons over ALF spaces [18]. These results fit into the original research plan.

In further directions, with Á. Nagy we calculated the partition function of Abelian gauge theory over ALF spaces via zeta-function regularization and heat kernel methods. This allowed us to obtain the transformation rule of the partition function under modular (i.e., S duality) transformations [16]. We gave a mathematical proof of a variant of the strong cosmic censor conjecture (SCCC) of R. Penrose [17]. Along these lines, making use of exotic $\mathbb{R}^4$'s we hope to find a generic counterexample for the SCCC in the near future. Finally, we gave a reformulation of general relativity in terms of algebraic quantum field theory. Representation theory of a resulting C^*-algebra led to a new modular functor i.e. a conformal field theory (CFT) in the sense of Segal [19].

2.14 Boldizsár Kalmár

In a joint work with Tamás Terpai, we provided a necessary condition for the existence of negative codimensional fold- and cusp-maps. Existence of a corank one map of negative codimension puts strong restrictions on the topology of the source manifold, see for example [30]. The paper [23] already appeared.

In a joint work with András Stipsicz, we constructed stable maps on 3-manifolds. We constructed a stable map from M to the plane, whose singular set is canonically oriented. We obtain upper bounds for the minimal numbers of crossings and non-simple singularities and of connected components of fibers of stable maps from M to the plane. Our paper was published in [24].

In another joint work with András Stipsicz, we show infinitely many examples of pairs of smooth, compact, homeomorphic 4-manifolds, whose diffeomorphism types are distinguished by the topology (i.e. the genus) of the singular sets of smooth stable maps defined on them. We apply a result of [29] in constructing maps with the desired properties on some of our examples. We appeal to Seiberg-Witten theory (in particular, to the adjunction inequality and its consequences) in showing that maps with certain prescribed singular sets do not exist on our carefully chosen other examples. The first and most obvious pair of examples for such phenomena is provided by [20]. We extend their idea to an infinite family of such exotic pairs. Our paper was published in [25].

Finally, we characterized those unions of embedded disjoint circles in the sphere S^2 which can be the multiple point set of a generic immersion of S^2 into $\mathbb{R}^3$ in terms of the interlacement of the given circles. Our result is the one higher dimensional analogue of Rosenstiehl's characterization of words being Gauss codes of self-crossing plane curves. The proof uses a result of [27] and further generalizes the ideas of [22], which leads to directed interlacement graphs of paired trees and their local complementation. The paper was submitted for publication.

2.15 Endre Szabó

A K*-approximate subgroup* of a group is a finite subset A such that A^3 is at most K-times larger than A. Helfgott's conjecture describes approximate subgroups in linear groups of bounded rank: A can be covered by K^m cosets of a nilpotent subgroup, where m depends on the rank of the linear group only. In a joint work with L. Pyber we reduced this conjecture to the case of solvable linear groups.

Suppose we are given a configuration of three families of curves in the plane, each having n curves in it. A *triple point* of the configuration is a point incident to a curve in each of the three families. The configuration is *rich* if it has at least Cn^2 triple points — it is a very special situation. With Elekes and Simonovits we have found some new geometric conditions which imply that the configuration is not rich.

2.16 Árpád Tóth

Together with Duke and Imamoglu we gave a description of Ramanujan mock modular functions, and showed that the Fourier coefficients of these functions can be given as integrals of classical modular functions along closed geodesics. This result generalizes a result of Borcherds. This result also provided a new proof of a famous result of Siegel. We also constructed modular integrals whose co-boundaries are rational period functions with poles at real quadratic integers. With Cojocaru we gave an approximation of the probability of cyclicity of the group of an ellpitic curve over a finite field with an extremely good error bound.

2.17 Ágnes Szilárd

Following groundbreaking work of J. Milnor in the 1960's, the topology of isolated complex hypersurface surface singularities is well-understood: it is a cone over the so-called link which is a three-manifold obtained by intersecting the nearby smooth fiber of the singularity with a sufficiently small ball. Moreover, 20 years later Walter Neumann showed that these three-manifolds can be described and are determined by a plumbing graph, thus giving a complete combinatorial characterization of these important topological manifolds.

Such description of non-isolated complex one-dimensional surface singularities was not available. In fact, unlike in the isolated case, here the boundary of the nearby smooth fiber (Milnor fiber) and of the singular fiber is different, making it impossible to generalize methods of the singular case. Because of this fundamental difference between the singular and non-singular cases, the topological study of the Milnor fiber in the non-singular case is very difficult.

In a joint work with Némethi — using completely novel methods — we succeeded in showing that in the non-isolated case as well, the boundary of the Milnor fiber can be given by plumbing graphs, obtaining a complete topological decription of such manifolds. We worked out an explicit, partially geometric, partially combinatorial algorithm for determining these graphs. As a byproduct, formulae for the various monodromies (horizontal, vertical), the corresponding Hodge structure has been given, and the method has been illustrated by a large number of examples. (These examples are typically rather tedious to work out.) The geometric construction in the algorithm revealed an inner structure of the singularities which opened new doors to study other aspects of non-isolated singularities such as the related open book decompositions and contact structure. The complete study was published in the Springer Lecture Notes Series, Vol. 2037 in 2012.

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