We wrote 191 scientific papers during the four years of the project and gave about 250 lectures on them on international conferences. It was possible only with the support of this OTKA project. We organized EUROCOMB11, the biggest European combinatorial conference, the chairman and the secretary of the program committee and the organizing committee were the supervisor and a participant of this project. We achieved very important results in the theory of graph limits, proved old extremal graph theory conjectures on trees (the proof is 160 pages!). We got deep and surprising extremal hypergraph theorems on cycles, partition critical hypergraphs. Beyond combinatorics, but by means of combinatorial tools we proved important Caratheodory type theorems in geometry, solved old problems of Erdos in number theory, characterized the positively finitely generated profinite groups and proved theorems related to bioinformatics or data bases.

As to the detailed scientific results, a very brief summary of some results is as follows.

We proved a central limit theorem about random polytopes when the starting body is a polytope.

We proved that every point on a closed 2-dimensional surface is a most distant point from some other point.

We determined the minimum circumference lattice polygon of n vertices for any norm.

We constructed sparse 3-uniform hamilton cycle saturated hypergraphs and estimated the smallest possible density for r-uniform hypergraphs. Similar results were proved for hamilton path saturated hypergraphs.

Instead of 1-factors, we studied f-parity factors in graphs. We proved a Gallai-Edmonds type structure theorem and by means of that, we proved theorems on barriers and elementary graphs related to f-parity factors.

The fractional chromatic number relaxations of several conjectures on chromatic number were proved earlier. We studied similar relaxations for the topological lower bound of the chromatic number.

We determined how many colors are needed to color conflict-freely the hypergraph obtained from the neighborhoods of the vertices of a graph.

We improved the estimate of the crossing number lemma in the case when the degrees are very far from each other.

We found systems of axis parallel rectangles such that any coloring with a given number of colors yields a point contain in another given number of rectangles of the same color.

We answered a question of the Fields medal winner Gowers by proving a new version of Jordans theorem about linear groups.

We proved several theorems about drawings of planar graphs. Especially, if the degrees ar bounded then the graphs can be drawn with few slopes.

We proved versions of Gyarfas tree packing conjectures if we have some restriction on most of the trees.

We proved the Havel-Hakimi inequality for directed graphs.

We have been studying the duality pairs in the homomorphism poset of finite (directed and/or undirected) graphs, as a special case of investigations of the splitting property in infinite posets. We gave a short, transparent proof of the seminal Foniok - Nesetril - Tardiff theorem on finite generalized duality pairs for finite directed graphs.

Erdos estimated the maximal number of integers selected from 1,2, ..., N so that no one of them divides the product of two others. We extended Erdos's problem to sets of integers such that no one of them divides the product of k others and by means of combinatorial results we estimated the size of such a set.

We proved that the amalgams of sofic groups over amenable groups are sofic. It was announced by Collins and Dykema, but the proof contains some mistake, it works only in some special case.

We proved that the Linnell-Schick regular closure is canonical in case of amenable groups. It is the structural version of the approxomation theorem of Luck.

We showed a 20 year old conjecture of van Douwen about amenable group actions.

We studied the properties of analogue group action, proved a conjecture of Lubotzky and extended a theorem of Shalom.

We proved a special case of Gyarfas' tree packing conjecture that if all but three of the trees are stars then they have a packing into any k-chromatic graph. We also consider several other generalizations of the conjecture.

We solved a path-search type problem of Soren Riis for complete t-ary trees. Also, for pyramid graphs we achieved some non-trivial partial results.

We proved that octants are cover-decomposable, i.e., any 12-fold covering of any subset of the space with a finite number of translates of a given octant can be decomposed into two coverings. As a corollary, we obtain that any 12-fold covering of any subset of the plane with a finite number of homothetic copies of a given triangle can be decomposed into two coverings. We also show that any 12-fold covering of the whole plane with the translates of a given open triangle can be decomposed into two coverings.

We prove lower and upper bounds for the chromatic number of certain hypergraphs defined by geometric regions. This problem has close relations to conflict-free colorings. One of the most interesting type of regions to consider for this problem is that of the axis-parallel rectangles. We completely solved the problem for the special case of bottomless rectangles. We also gave an almost complete answer for half-planes.

A strong box-respecting coloring of an n-dimensional box partition is a generalization of rectangle-respecting colorings and strong polychromatic colorings to more than 2 dimensions. A guillotine-partition is obtained by starting with a single axis-parallel box and recursively cutting a box of the partition into two boxes by a hyperplane orthogonal to one of the n coordinate axes. We proved that there is a strong box-respecting coloring of any n-dimensional guillotine-partition.

We considered right angle crossing (RAC) drawings of graphs in which the edges are represented by polygonal arcs and any two edges can cross only at a right angle. We show that if a graph with n vertices admits a RAC drawing with at most 1 bend or 2 bends per edge then the number of edges is at most 6.5n and 74.2n, respectively. This is a strengthening of a recent result of Didimo et al.

We investigated the following search problem. Given an n element set with at least k faulty elements. In one query we can ask a subset A and the answer is yes if and only if the ratio of faulty elements in A is at least a fixed number. The aim is to find m faulty elements. We gave several lower and upper bounds on the number of queries needed.

By a polygonisation of a finite point set S in the plane we understand a simple polygon having S as the set of its vertices. Let B and R be sets of blue and red points in general position, that the convex hull of B contains k interior blue points and l interior red points. Hurtado et al. found sufficient conditions for the existence of a blue polygonisation that encloses all red points. We considered the dual question of the existence of a blue polygonisation that excludes all red points R. We showed that there is a minimal number K, which is polynomial in l, such that one can always find a blue polygonisation excluding all red points, whenever k¿K.

We gave a Markov Chain Monte carlo method to investigate the dynamics of vegetation maps statistically.

We investigated the local chromatic number of shift graphs and proved that it is close to their chromatic number. This implies that the gap between the directed local chromatic number of an oriented graph and the local chromatic number of the underlying undirected graph can be arbitrarily large.

We axiomatized functional dependencies and weak functional dependencies in complex values datamodels that give theoretical foundations to such widespread constructs as XML- databases.

We continued the work begun by Anstee, Ferguson, Griggs, Kamoosi and Sali on (0,1)-matrices with small forbidden configurations. We compute exact bounds for all 4x2 forbidden configurations.

We presented a distance measure for databases based on the poset of closures, i.e., how much the sets of functional dependencies satisfied by the databases differ from each other. Some extremal combinatorial properties of this distance are showed.

Suppose there is an unknown number of defective elements in an n-element set, but we want to find only one or to claim that there is none. We can use subsets asking if they contain at least one defective. It is proved that the number of questions (subsets) is at least n. For two rounds the fastest method is determined.

Suppose we have a family of m-subsets of an n element set. If m is big then there is at least one pair of disjoint subsets. The probability of the event that omitting the members randomly with probability p an intersecting family is obtained. Its maximum for all families with m members is is determined for some m.

We gave a simple proof of a Waring type theorem of Shalev on simple groups.

We characterized the positively finitely generated profinite groups proving a conjecture of Lubotzky, Mann, Segal and others.

We proved many results in extremal combinatorics and related subjects. In this report we select just a few of them to present.

We proved a Product Theorem proved independently by Breuillard, Green and Tao about the minimum number of generators of a Lie type simple group.

Secret sharing has a strong connection to combinatorics, especially linear programming. We found some methods and ideas where LP-problems occur and presented the resulting estimations in a special case.

We determined the minimum size of graphs that contains a path on 4 distinct vertices even after the removal of any edge.

We have been studying the duality pairs in the homomorphism poset of finite (directed and/or undirected) graphs, as a special case of our general investigations of the splitting property in infinite posets. Our newest research paper gives a short, transparent proof of the seminal Foniok - Nesetril - Tardiff theorem on finite generalized duality pairs for finite directed graphs, and also shows that there exists no finite – infinite duality pair in the same poset.

After at least four year research on more-part Sperner families two papers were published: in the first paper, we proved some important, fundamental properties of these structures. Finally using this result we constructed a new family of mixed orthogonal arrays.

We enumerated multi-labeled trees: these are phylogenetic trees, where the leaves (the existing species whose evolution we want to model) bear several labels, coming from (possibly common) genes.

We found the right setup to state (and prove) strong Sperner and BLYM type results for normal posets. The expression strong refers to the fact that in case of equality the extremal systems are homogeneous. The possibly most complicated result here is a strong two-part Sperner theorem for direct product of two normal, unimodal posets.

It is always interesting to define and solve other two-part type combinatorial problems. The first real hardness here is to come up with meaningful two-part problem. We find at least three possible two-part problem classes: like two-part intersection; one-part intersection – one-part Sperner; finally two-part intersection – two-part Sperner families. One problem is fully solved while for the other classes we found reasonable lower and upper bounds.

We studied an - originally data mining - problem which is closely related to periodic substring finding. Here we are given a large underlying set (for example mobile-phone owners) as possible vertices of large graphs, and we have snapshots of interaction among them - they are represented by edges of the graph. Each studied separated time-slot corresponds to one particular graph -and are interested maximal edge subsets with periodic behavior within this time series.

We continued our research in several areas of bioinformatics, especially on sequences in genoms.

We extended Jarniks result of 1926 about convex lattice polygons for non-symmetric norms and proved a limit form theorem too.

We proved assymptotically tight upper bounds on the variance of probability variables defined on random polygons.

We gave assymptotically tight lower bounds on the logarithm of the number of equivalence classes of d dimensional convex lattice polytopes of given volume.

By means of algebraic geometry, tensors and combinatorics, we proved several extensions of Caratheodorys famous theorem.

We determined which is the family of sets that contains $2^{n-1} + i$ sets and when the sets are chosen independently with probability p, then the probability of obtaining an intersecting family is as high as possible. We have results for the analougus question in k-uniform families.

We investigated an extension of the pebbling problem. In rubbling one can remove 2 pebbles from a neighbor of a vertex, and place one of them to this vertex. We determined the maximum of the optimal rubbling number in a diameter 2 graph up to a small additive constant.

We disproved the conjecture of Grigorchuk and Medynets for Z2 systems that the topological full group of a minimal Z system is amenable.

We found and described the connection between hyper finite graph classes and amenability.

We presented a distance measure for databases based on the poset of closures, i.e. how much the sets of functional dependencies satisfied by the databases differ from each other.

We gave general extension of the celebrated Sauer, Perles and Shelah, Vapnik and Chervonenkis result from 0-1 sequences to k-ary codes still giving a polynomial bound.

We studied a new, generalized version of the well-known group testing problem finding at least one defective elemen tor all of them. In our model the presence of defective elements in a test set Q can be recognized if and only if their number is large enough compared to the size of Q.

We settled a problem of Dujmovic, Eppstein, Suderman, and Wood by showing that there exists a function f with the property that every planar graph G with maximum degree d admits a drawing with non-crossing straight-line edges, using at most f(d) different slopes. If we allow the edges to be represented by polygonal paths with one bend, then 2d slopes suffice. Allowing two bends per edge, every planar graph with maximum degree d¿2 can be drawn using segments of at most [(d+1)/2] different slopes.

We considered right angle crossing (RAC) drawings of graphs in which the edges are represented by polygonal arcs and any two edges can cross only at a right angle. We showed that if a graph with n vertices admits a RAC drawing with at most 1 bend or 2 bends per edge, then the number of edges is at most 6.5n and 74.2n, respectively. This is a strengthening of a recent result of Didimo et al.

We proved lower and upper bounds for the chromatic number of certain hypergraphs defined by geometric regions. This problem has close relations to conflict-free colorings. One of the most interesting type of regions to consider for this problem is that of the axis-parallel rectangles. We completely solve the problem for a special case of them, for bottomless rectangles. Moreover, we gave efficient coloring algorithms.

We proved that octants are cover-decomposable, i.e. any 12-fold covering of any subset of the space with a finite number of translates of a given octant can be decomposed into two coverings.

We proved a theorem on dominating sets in digraphs and applied it to settle a problem related to generalized Gallai colorings, that are edge colorings of graphs without 3-colored triangles.

We have been studying the duality pairs in the homomorphism poset of finite (directed and/or undirected) graphs, as a special case of our general investigations of the splitting property in infinite posets. We tried to prove the conjecture that there is no infinite-finite duality pair in the poset of finite directed graphs. Instead we actually proved there are such duality pairs. Finally, we developed a full theory to generate all such pairs, and found their complete characterization.

We used logarithmic space sampling algorithms to solve various streaming problems optimally. Among them is the problem of finding duplicates: we present an algorithm that uses logarithmic space to find a duplicate with high probability in a data stream of n+1 objects from a universe of size n.

Secret sharing is a cryptographic primitive: shares of a secret has to be distributed among players such that certain qualified subsets of the player are able to reconstruct the secret while unqualified subsets learn nothing about it. We explored the on-line version: players arrive one by one and receive their share. The distributor is only informed about the qualified subsets of the players that have arrived.

We call a plane drawing of a graph k-locally plane if crossing edges are at least k apart in the graph. We showed that for arbitrary fixed value of k there exist k-locally plane graphs with a superlinear number of edges.

We proved a slight sharpening of the Erdos-Ko-Rado theorem for graphs.

We applied our probabilistic results for random databases. Given the number of attributes (columns) and the individuals (rows) in a database (matrix), choose the entries randomly, where the rows are independent, but the columns can be dependent. Knowing the joint distributions, We determined the typical sizes of keys (dependencies) asymptotically.

The minimum of the average number of questions is determined in the 1 search problem where the classes in the question partitions have size at most k.

We proved the old conjecture of Weiss on G-locally primitive graphs for graphs with automorphism groups admitting composition factors of bounded rank.

We continued our research on limits of graph sequences especially on convergent sequences of dense graphs.

We found a new proof of the hypergraph regularity and removal lemmas and for testing hypergraph properties by means of set theoretical tools. We constructed the limit of convergent hypergraph sequences.

We proved further interesting cases of the famous tree packing conjecture. In another paper, we considered generalizations of this conjecture and proved that if all but three of the trees are stars then they have a packing into any k-chromatic graph.

We determined the extremal number and found the unique extremal graph for forests consisting of paths or stars.

We found the complexity of the biased Maker-Breaker game to build an odd cycle.

We studied the problem of finding the minimum size of a family saturating the k-Sperner property and the minimum size of a family that saturates the Sperner property and that consists only of l-sets and (l+1)-sets.

We investigated the largest possible sizes of almost cross-intersecting and almost cross-Sperner pairs of families of sets.

We determined the irregularity strength s(G) and vertex irregularity strength tvs(G) of circulant graphs.=20

We consider the problem having n points on the plane and we want to find a polygonal path which contains all of them such that the number of segments is as small as possible. We proved lower and upper bounds for this number.

We are given n colored balls and a query shows if two balls have the same color or not. We investigated the problem to find a ball from the largest color class using as few queries as possible.

We proved the bounded rank case of the Liebeck-Nikolov-Shalev conjecture about product decompositions of simple groups.

It is well-known, that any two realizations of a given degree sequence can be transformed into each other using sequence of swap operations. We determined the shortest possible length of this sequence - unfortunately, by means of other graph parameters.

We introduced a new class of degree sequence problems: we want to find graphical realizations of a given degree sequence on labeled vertices, where certain would-be edges are forbidden. We solved this restricted degree sequence problem if the forbidden edges form a bipartite graph consisting of the union of a matching and a star.

We proved the following version of the Loebl-Komlos-Sos Conjecture: For every c¿0 there exists a number M such that for every k¿M, every n-vertex graph G with at least (0.5+c)n vertices of degree at least (1+c)k contains each tree T of order k as a subgraph.The method to prove our result follows a strategy common to approaches which use the Szemeredi Regularity Lemma: we decompose the graph G, find a suitable combinatorial structure inside the decomposition, and then embed the tree T into G using this structure. However, the decomposition given by the Regularity Lemma is not of help when G is sparse. To surmount this shortcoming, we use a more general decomposition technique: each graph can be decomposed into vertices of huge degree, regular pairs (in the sense of the Regularity Lemma), and two other objects each exhibiting certain expansion properties. The final version of the paper is 160(!) pages, so it will not be easy to publish it, even if this is the proof of a famous conjecture.

We extended the following graph theory result of Erdos and Gallai for hypergraphs: What is the maximum number of edges in a graph that does not contain a path of length k? In hypergraphs, paths can be generalized in many different ways. We gave sharp upper and lower bounds in most cases.

Partition critical hypergraphs are generalizations of 3-critical hypergraphs. We proved upper bounds on the size of them and generalized a theorem of Lovasz. We also introduced the concept of ordered-3-critical hypergraphs and gave constructions that show that partition criticality is a weaker condition than ordered-3-critical or 3-critical.

The local chromatic number is an interesting coloring parameter of graphs that lies between the fractional chromatic number and the chromatic number. This suggests to investigate it for graphs for which the interval between the mentioned two parameters is large. It turns out that some of these graphs quadrangulate certain non-orientable surfaces. Investigating the chromatic properties of such quadrangulations is a living area of research. We made such investigations for the local chromatic number of these quadrangulating graphs. It turned out, that, somewhat surprisingly, the behavior of the local chromatic number (unlike that of the chromatic number) highly depends on the genus of the given surface.

We determined or bounded asymptotics of the maximum cardinality of a set of Hamiltonian paths in the complete graph such that the union of any two paths from the family contains a cycle of some length from a given set of integers.

We proved various Helly-type and Hadwiger-Debrunner type theorems for coloured systems of convex bodies.

We proved that there are many (namely of order n/log n) empty triangles sharing a common edge among n uniformly and independently chosen points from a planar convex set. It is a partial solution of a more general conjecture about arbitrary (but general position) point sets in the plane.

We showed several new and deep Caratheodory type results using geometric, topological, and combinatorial methods.

We showed that every point is critical in an Alexandrov space, i.e., it is the farthest point from some point of the space.

Secret sharing is a cryptographic primitive where players are given certain ”shares” of a secret such that a qualified set of players can recover the secret but an unqualified set of players get no information about the secret. The goal is to make the shares small compared to the complexity of the secret. We studied a novel problem when the shares have to be assigned to the players as they show up without perfect knowledge of their role and the qualified sets.

A geometric graph is a graph drawn in the plane possibly with intersections. It is k-locally plane if each path of length k is intersection free. We proved for arbitrary fixed k, that there are k-locally plane graphs with the number of edges super-linear in the number of vertices.

A quasi-rectangle is a the set of points in the plane that is swept by an interval that is moved in a (possibly changing) direction, always almost perpendicular (between 89 and 91 degrees) to the interval. We prove that the smallest set of points that hit every quasi-rectangle that lies within the unit square and has area at least 1/n is of nlog n.