Dynamical and Geometric Aspects of Measure theory
OTKA K 75242 Project
Closing Report

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Summary

This project had one participant and it was extended twice by one year and hence lasted for six years.

In dynamical systems and ergodic theory we made progress in the following areas:

- almost everywhere convergence of nonconventional ergodic averages, especially Bourgain's question related to convergence along the squares,
- a question of J. P. Conze about existence of maximizing points,
- size of rotation sets sufficiently large to guarantee the the function considered should be in L^1,
- obtained so far unpublished partial results concerning equientropy curves of skew tent maps in the square.

In geometric measure theory and real functions:

- singularity of the occupation measure of the Weierstrass-Cellerier function,
- multifractal properties of time subordinations of real functions,

- multifractal properties of generic/typical measures and of functions monotone in several variables,
- homogeneous multifractal measures with prescribed multifractal spectrum,
- introduced a new concept, the topological Hausdorff dimension and in two papers investigated its properties.
- obtained so far unpublished partial results concerning fractal percolation.

11 papers with various coauthors were published during the grant period. One of them in the top journal Annals of Mathematics, but the other journals in which we published include Nonlinearity, Journal of Fourier Analysis and Applications, Israel Journal of Mathematics, Ergodic Theory and Dynamical Systems, Chaos Solitons and Fractals, Acta Mathematica Hungarica, Journal of Fractal Geometry and Journal of Mathematical Analysis and Applications. There are three more papers on the publication list one of them is accepted for publication Ergodic Theory and Dynamical Systems, another, after a positive referee's report is in the final stage of acceptance (revision after a positive referee's report) in the high prestige Advances in Mathematics.
The third one is only submitted and the preprint is available from my home page.

I listed 14 conference lectures, out of them 5 were invited talks. I gave 9 departmental seminar talks abroad about my results. I also gave two mini-courses of several lectures one in the USA and the other one in Africa.

Remarks I attached to this report information about ten published papers from the Hungarian data base MTMT. One published paper, [11] is missing from this list, since it is published in the first volume of a new periodical Journal of Fractal Geometry
`http://www.ems-ph.org/journals/journal.php?jrn=JFG`
this journal published by the European Mathematical Society and is already in the German Zentralblatt database but not yet in Mathscinet, or the MTMT journal database. Anyway, as one looks at the editorial board it is clear that we should feel honored that, on the request of Kenneth Falconer, my coauthor sent our paper to this journal.

It takes a while until a paper gets published and it also takes for a while until work giving reference to a paper is carried out and published, hence for short duration grants one cannot expect too much in the MTMT data. This grant was extended to a 6 year grant and hence these MTMT data are more relevant, especially for papers published at the beginning of the grant period.

Scientific Results

DYNAMICAL SYSTEMS AND ERGODIC THEORY:

In [5] we answer a question of J. Bourgain which was motivated by questions of A. Bellow and H. Furstenberg. We show that the sequence $\{n^2\}_{n=1}^{\infty}$ is L^1-universally bad. This implies that it is not true that given a dynamical system (X, Σ, μ, T) and $f \in L^1(\mu)$, the ergodic means

$$\lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} f(T^{n^2}(x))$$

converge almost surely.

This paper (after six years of "struggle") finally got published in the extremely high prestige Annals of Mathematics. As one looks at the citation list of this paper in the attached MTMT list one can see that there was quite an international interest in this result by leading scientists of the field.

In January of 2013 I was invited by Fedor Nazarov, Artem Zvavitch and Dmitry Ryabogin to Kent State university in Ohio to give a one week mini-course of five 90 minute lectures about this paper. I think it should be emphasized that this group of researchers was not much known for me previously and the base of invitation was their interest in the, quite intricate details of this paper.

A continuation of this project about the divergent square averages is the following result. In [10], answering a question raised by J-P. Conze we show that for any $x, \alpha \in \mathbb{T}$, $\alpha \notin \mathbb{Q}$ there exist $f \in L^1(\mathbb{T})$, $f \geq 0$ such that the averages

$$(\star) \qquad \frac{1}{N} \sum_{n=1}^{N} f(y + nx + n^2\alpha)$$

diverge for a.e. y. By Birkhoff's Ergodic Theorem applied on $\mathbb{T}^2$ for the transformation $(x, y) \mapsto (x + \alpha, y + 2x + \alpha)$ for almost every $x \in \mathbb{T}$ the averages $(\star)$ converge for a.e. y. We show that given $\alpha \notin \mathbb{Q}$ one can find

$f \in L^1(\mathbb{T})$ for which the set $D_{\alpha,f} \stackrel{\text{def}}{=} \{x \in \mathbb{T} : (\star)$ diverges for a.e. y as $N \to \infty\}$ is of Hausdorff dimension one. We also show that for a polynomial $p(n)$ of degree two with integer coefficients the sequence $p(n)$ is universally L^1-bad.

Methods of our divergent square averages paper initiated a cooperation with I. Assani related to Bourgain's return times property of the tail question. We continue our work in this direction in [3]. Let $(X, \mathcal{B}, \mu, T)$ be an ergodic dynamical system on a finite measure space. Consider the maximal function $R^* : (f, g) \in L^p \times L^q \to R^*(f, g)(x) = \sup_n \dfrac{f(T^n x) g(T^{2n} x)}{n}$. We prove that if p and q are greater or equal than one and $\frac{1}{p} + \frac{1}{q} < 2$ then R^* maps $L^p \times L^q$ into any L^r as long as $0 < r < 1/2$. This implies that $R^*(f, g)$ is finite almost everywhere and $\frac{f(T^n x) g(T^{2n} x)}{n} \to 0$ for a.e. x as $n \to \infty$.

Questions of J. P. Conze motivated the joint paper [9] with J. Bremont. Consider an irrational rotation of the unit circle and a real continuous function. A point is declared "maximizing" if the growth of the ergodic sums at this point is maximal up to an additive constant. In case of two-sided ergodic sums the existence of a maximizing point for a continuous function implies that it is the coboundary of a continuous function. In contrast, we build for the "usual" one-sided ergodic sums examples in Hölder or smooth classes indicating that all kinds of behaviour of the function with respect to the dynamical system are possible. We also show that generic continuous functions are without maximizing points, not only for rotations, but for the transformation $2x \mod 1$ as well. For this latter transformation it is known that any Hölder continuous function has a maximizing point.

In the next two papers I returned to one of my old questions about rotation sets. Suppose that $f : \mathbb{R} \to \mathbb{R}$ is a given measurable function, periodic by 1. For an $\alpha \in \mathbb{R}$ put $M_n^\alpha f(x) = \dfrac{1}{n+1} \sum_{k=0}^{n} f(x + k\alpha)$. Let Γ_f denote the set of those α's in $(0, 1)$ for which $M_n^\alpha f(x)$ converges for almost every $x \in \mathbb{R}$. We call Γ_f the rotation set of f. We proved earlier that if Γ_f is of positive Lebesgue measure then f is integrable on $[0, 1]$, and hence, by Birkhoff's Ergodic Theorem all $\alpha \in [0, 1]$ belongs to Γ_f. However, $\Gamma_f \backslash \mathbb{Q}$ can be dense (even c-dense) for non-L^1 functions as well. In [4] we answer a more than a decade old question by showing that there are non-L^1 functions for which Γ_f is of Hausdorff dimension one.

With my Ph. D. student G. Keszthelyi in [13] we continued this direction of research in a bit more abstract algebraic setting. Suppose that G is a compact Abelian topological group, m is the Haar measure on G and $f : G \to \mathbb{R}$ is a measurable function. Given (n_k), a strictly monotone increasing sequence of integers we consider the nonconventional ergodic/Birkhoff averages

$$M_N^\alpha f(x) = \frac{1}{N+1} \sum_{k=0}^{N} f(x + n_k\alpha).$$

The f-rotation set is

$\Gamma_f = \{\alpha \in G : M_N^\alpha f(x)$ converges for m a.e. x as $N \to \infty.\}$

We prove that if G is a compact locally connected Abelian group and $f : G \to \mathbb{R}$ is a measurable function then from $m(\Gamma_f) > 0$ it follows that $f \in L^1(G)$.

A similar result is established for ordinary Birkhoff averages if $G = Z_p$, the group of p-adic integers.

However, if the dual group, $\widehat{G}$ contains "infinitely many multiple torsion" then such results do not hold if one considers non-conventional Birkhoff averages along ergodic sequences.

What really matters in our results is the boundedness of the tail, $f(x + n_k\alpha)/k$, $k = 1, ...$ for a.e. x for many α, hence some of our theorems are stated by using instead of Γ_f slightly larger sets, denoted by $\Gamma_{f,b}$.

Geometric Measure theory and real functions:

Paper [1] is a continuation of my project about micro tangent sets and irregular one sets on the graphs of continuous functions. In this paper we show that the occupation measure of the Weierstrass–Cellerier function $\mathcal{W}(x) = \sum_{n=0}^{\infty} 2^{-n} \sin(2\pi 2^n x)$ is purely singular. Using our earlier results previously supported by other OTKA grants we can deduce from this that almost every level set of $\mathcal{W}(x)$ is finite. These previous results and Besicovitch's projection theorem imply that for almost every c the occupation measure of $\mathcal{W}(x, c) = \mathcal{W}(x) + cx$ is purely singular. In this paper we verify that this result holds for *all* $c \in \mathbb{R}$, especially for $c = 0$. As it happens quite often it is not that easy to obtain from an almost everywhere true statement one that holds everywhere.

The next five papers mentioned are results of my cooperation with Stéphane Seuret.

A question of Yves Meyer motivated the research concerning "time" subordinations of real functions. Denote by Lip $^\alpha$ the metric space of functions with Lipschitz constant 1 defined on $[0, 1]$, equipped with the complete metric defined via the supremum norm. Given a function $g \in \text{ Lip }^{\alpha}$ one obtains a time subordination of g simply by considering the composite function $Z = g \circ f$, where $f \in \mathcal{M} := \big\{ f : f(0) = 0,\ f(1) = 1$ *and* f *is a continuous nondecreasing function on* $[0,1] \big\}$. The metric space $\mathcal{E}^{\alpha} = \mathcal{M} \times \text{ Lip }^{\alpha}$ equipped with the product supremum metric is a complete metric space. In [7] for all $\alpha \in [0,1)$ multifractal properties of $g \circ f$ are investigated for a generic (typical) element $(f,g) \in \mathcal{E}^{\alpha}$. In particular, we determine the generic Hölder singularity spectrum of $g \circ f$.

In [2] we prove that in the Baire category sense, measures supported by the unit cube of $\mathbb{R}^d$ typically satisfy a multifractal formalism. To achieve this, we compute explicitly the multifractal spectrum of such typical measures μ. This spectrum appears to be linear with slope 1, starting from 0 at exponent 0, ending at dimension d at exponent d, and it indeed coincides with the Legendre transform of the L^q-spectrum associated with typical measures μ.

In [6] we study the singularity (multifractal) spectrum of continuous functions monotone in several variables. We find an upper bound valid for all functions of this type, and we prove that this upper bound is reached for generic functions monotone in several variables. Let E_f^h be the set of points at which f has a pointwise exponent equal to h. For generic monotone functions $f : [0,1]^d \to \mathbb{R}$, we have that $\dim E_f(h) = d - 1 + h$ for all $h \in [0,1]$, and in addition, we obtain that the set E_f^h is empty as soon as $h > 1$. We also investigate the level set structure of such functions.

In [11] we construct measures supported in $[0,1]$ with prescribed multifractal spectrum. Moreover, these measures are homogeneously multifractal (HM, for short), in the sense that their restriction on any subinterval of $[0,1]$ has the same multifractal spectrum as the whole measure. The spectra f that we are able to prescribe are suprema of a countable set of step functions supported by subintervals of $[0,1]$ and satisfy $f(h) \leq h$ for all $h \in [0,1]$. We also find a surprising constraint on the multifractal spectrum of a HM measure, that we call Darboux theorem for multifractal spectra of measures: the support of its spectrum within $[0,1]$ must be an interval. Using wavelet theory, we also build HM functions with prescribed multifractal spectrum.

Due to publication length restrictions a technical part of the more than 30 pages long paper [11] was separated into an independent paper [14] which is at this time submitted for publication elsewhere.
We proved in [11] that the support of the multifractal spectrum of a homogeneously multifractal (HM) measure within $[0,1]$ must be an interval. In [14] we construct a homogeneously multifractal measure with spectrum sup-

ported by $[0,1] \cup \{2\}$. This shows that there can be a different behaviour for exponents exceeding one. We also provide details of the construction of a strictly monotone increasing monohölder (and hence HM) function which has exact Hölder exponent one at each point.

Papers [8] and [12] grew out from a problem which I submitted to the 2008 annual Schweitzer competition:

Suppose S is the Sierpinski triangle. For the generic continuous function on S determine the Hausdorff dimension of the level sets $f^{-1}(y)$.

Answer: $\dim f^{-1}(y) = 0$ for all $x \in f(S)$.

Originally I submitted two versions of the problem to the competition committe. A more difficult version, not accepted by the committee was the following:

Suppose S is the Sierpinski carpet. For the generic continuous function on S determine the Hausdorff dimension of the level sets $f^{-1}(y)$.

Answer: $\dim f^{-1}(y) = \log 2/\log 3$ for all $x \in \mathrm{int} f(S)$, and $f^{-1}(y)$ consists of one point if $y \in \{\max f(S), \min f(S)\}$.

R. Balka, at that time a graduate student, got interested in the problem and this has become a joint research project with him and his advisor M. Elekes.

In [12] we introduce a new concept of dimension for metric spaces, the so called *topological Hausdorff dimension*. It is defined by a very natural combination of the definitions of the topological dimension and the Hausdorff dimension. The value of the topological Hausdorff dimension is always between the topological dimension and the Hausdorff dimension, in particular, this new dimension is a non-trivial lower estimate for the Hausdorff dimension.

We examine the basic properties of this new notion of dimension, compare it to other well-known notions, determine its value for some classical fractals such as the Sierpiński carpet, the von Koch snowflake curve, Kakeya sets, the trail of the Brownian motion, etc.

As our first application, we generalize the celebrated result of Chayes, Chayes and Durrett about the phase transition of the connectedness of the limit set of Mandelbrot's fractal percolation process. They proved that certain curves show up in the limit set when passing a critical probability, and we prove that actually 'thick' families of curves show up, where roughly speaking the word thick means that the curves can be parametrized in a natural way by a set of large Hausdorff dimension. The proof of this is basically a lower estimate of the topological Hausdorff dimension of the limit set. For the sake of completeness, we also give an upper estimate and conclude that in the non-trivial cases the topological Hausdorff dimension is almost surely strictly below the Hausdorff dimension.

Finally, as our second application, we show that the topological Hausdorff dimension is precisely the right notion to describe the Hausdorff dimension of the level sets of the generic continuous function (in the sense of Baire category) defined on a compact metric space.

Paper [8] deals with the following. For a compact metric space K let $\dim_H K$ and $\dim_{tH} K$ denote its Hausdorff and topological Hausdorff dimension, respectively. We proved in [12] that this new dimension describes the Hausdorff dimension of the level sets of the generic continuous function, that is $\sup\{\dim_H f^{-1}(y) : y \in \mathbb{R}\} = \dim_{tH} K - 1$ for the generic $f \in C(K)$. We also proved in [12] that if K is sufficiently homogeneous then $\dim_H f^{-1}(y) = \dim_{tH} K - 1$ for the generic $f \in C(K)$ and the generic $y \in f(K)$. The most important goal of [8] is to make these theorems more precise.

As for the first result, we prove that the supremum is actually attained, and also show that there may only be a unique level set of maximal Hausdorff dimension.

As for the second result, we characterize those compact metric spaces for which for the generic $f \in C(K)$ and the generic $y \in f(K)$ we have $\dim_H f^{-1}(y) = \dim_{tH} K - 1$. We also generalize a result of B. Kirchheim by showing that if K is self-similar then for the generic $f \in C(K)$ for every $y \in \mathrm{int} f(K)$ we have $\dim_H f^{-1}(y) = \dim_{tH} K - 1$.

Finally, we prove that the graph of the generic $f \in C(K)$ has the same Hausdorff and topological Hausdorff dimension as K.

Still in progress:

Our papers with Ph. D. student G. Keszthelyi concerning “Equi-kneading of skew tent maps in the square” are still not in their final form. Unfortunately, some parameter estimates turned out to be more difficult than expected and this time there is still some extra work needed to simplify the very technical and lengthy computations to obtain a publishable result.

Recently we also started a new research project related to fractal percolation with Maarit and Esa Järvenpää from Oulu, Finland. The paper containing our results is still in preparation.

Conference lectures related to this project:

1. “A conference in ergodic theory Dynamical Systems and Randomness” May 2009, Paris, Institut Henri Poincaré, invited talk, title: “Divergent square averages”.

2. “Ergodic Theory Workshop” University of North Carolina, April 2010, invited talk: “Averages along the squares on the torus”.

3. I was scheduled to give a talk with the title “Level set structure of real functions” as an invited speaker at the “International Conference on Mathematical Analysis 2010”, in Bangkok, Thailand but due to the political situation the conference at the last minute was cancelled. Instead, I gave this talk at our own ELTE Department of Analysis Seminar.

4. “Dynamical Systems and Dimension Theory Workshop” April 2011 Warwick, United Kingdom, 50 minute talk: “Non-L1 functions with rotation sets of Hausdorff dimension one”.

5. “Ergodic Theorems, Group Actions and Applications” May 2011, Eilat, Israel 40 minute talk: “Maximizing points and coboundaries for an irrational rotation on the Circle”.

6. “25th Summer Symposium in Real Analysis,” June 2011 contributed talk: “Hölder spectrum of functions monotone in several variables”.

7. “Fractals and Related Fields 2” Porquerolles (France), June 2011, contributed talk: “Hölder spectrum of functions monotone in several variables”.

8. “Ergodic Theory and Dynamical Systems: Perspectives and Prospects” Warwick, (United Kingdom) April 2012, contributed talk: “Equi-kneading of skew tent maps in the square”.

9. “Logic, Dynamics and Their Interactions, with a Celebration of the Work of Dan Mauldin” University of North Texas, Denton June 2012, “Special session” invited 30 minute talk: “Equi-kneading of skew tent maps in the square”.

10. “XXXVI. Summer Symposium in Real Analysis” Penn State University (Berks, USA), June 2012, 60 minute invited talk:“Level set structure of real functions”.

11. “Erdős Centennial” Budapest, July 2013, 30 minute invited talk: “Divergent square averages and related topics”.
12. “Visegrad Conference on Dynamical Systems” Olsztyn, Poland September 2013, 50 minute talk: “Equientropy curves of skew tent maps in the square”.
13. “ERC Workshop on Geometric Measure Theory, Analysis in Metric Spaces and Real Analysis” Scuola Normale Superiore, Pisa (Italy), September 2012, 50 minute invited talk: “Measures and functions with prescribed homogeneous multifractal spectrum”.
14. “International Conference on Fractal Geometry and Stochastics V,” March 2014, Tabarz (Germany) talk title: “Averages along the squares on the torus”.

Seminar talks abroad related to this project:

1. “Université Paris-Est Créteil Val de Marne” October 2009, departmental seminar 60 minute talk “Generic level sets with fractal properties”.
2. “Université Paris-Est Créteil Val de Marne” October 2010, departmental seminar 60 minute talk “On the gradient problem”.
3. “Université de Rennes I” January 2011, departmental seminar 50 minute talk ”Maximizing points and coboundaries for an irrational rotation on the Circle”.
4. “Séminaire Cristolien d'Analyse Multifractale” held at ”Université Paris-Est Créteil Val de Marne” January 2012, 60 minute invited talk: “Topological Hausdorff Dimension”.
5. “University of Helsinki” October 2012, two 45 minute departmental seminar talks “Multifractal Hölder spectra of generic functions and measures”.
6. “University of Oulu” October 2012, 60 minute departmental seminar talk “Ergodic averages along the squares and related questions”.

7. Kent State University (Ohio, USA), January 2013, 60 minute departmental seminar talk “Maximizing points and co-boundaries for an irrational rotation on the Circle”.
8. “Séminaire Cristolien d’Analyse Multifractale” ”Université Paris-Est Créteil Val de Marne”, October 2013, 60 minute invited talk: “ The Haight-Weizsäcker problem”.
9. University of Oulu, August 2014, 60 minute departmental seminar talk “On the gradient problem of C. E. Weil”.

Mini-courses:

1. January 2013, Kent State University (Ohio, USA) mini-course of five 90 minute lectures with the title: “Divergent Square Averages”.
2. During June of 2014 I was invited to be one of the five professors to give a course (four 90 minute lectures plus discussion sessions) at the two week long “ICTP-NLAGA School in Dynamical Systems and Ergodic Theory” held in Mbour, Senegal. (The other four professors of this school were Jose F. Alves, Christian Bonatti, Stefano Luzzatto and Marcelo Viana.)

Research cooperation:

As part of my research cooperation with I. Assani we published the paper [3].

With the group of researchers in Paris (S. Seuret and J. Bremont) we prepared and published papers [2], [6], [7], [9] and [11].

I also started a new international research project with Maarit and Esa Järvenpää from Oulu, Finland.

Foreign visitors in Budapest for short research visit sponsored by this project

S. Seuret two visits in 2009 and 2010.

Maarit and Esa Järvenpää 2013.

Short, one or two week, research visits abroad

Université Paris-Est Créteil Val de Marne in years 2009, 2010, 2012.

Université de Rennes I in 2011.

University of Oulu in 2014.

References

[1] Buczolich Zoltán: Occupation measure and Level Sets of the Weierstrass-Cellerier Function, Recent Developments in Fractals and Related Fields, editors J. Barral and S. Seuret, 3-18., 2010.

[2] Buczolich Zoltán and Seuret Stéphane: Typical Borel measures on $[0,1]^d$ satisfy a multifractal formalism., NONLINEARITY 23:(11) pp. 2905-2911., 2010.

[3] I. Assani and Z. Buczolich: The (L^p, L^q) bilinear Hardy-Littlewood function for the tail, ISRAEL JOURNAL OF MATHEMATICS 179:(1) pp. 173-187. (2010).

[4] Z. Buczolich: Non-L^1 functions with rotation sets of Hausdorff dimension one, ACTA MATHEMATICA HUNGARICA 126:(1-2) pp. 23-50. (2010).

[5] Z. Buczolich and D. Mauldin: Divergent Square Averages, ANNALS OF MATHEMATICS 171:(3) pp. 1479-1530., 2010.

[6] Z Buczolich and S. Seuret: Multifractal spectrum and generic properties of functions monotone in several variables., JOURNAL OF MATHEMATICAL ANALYSIS AND APPLICATIONS 382: (1) pp. 110-126., 2011.

[7] Z. Buczolich and S. Seuret: Singularity Spectrum of Generic a-Hölder Regular Functions After Time Subordination, JOURNAL OF FOURIER ANALYSIS AND APPLICATIONS 17: (3) pp. 457-485., 2011.

[8] R. Balka, Z. Buczolich and M. Elekes: Topological Hausdorff dimension and level sets of generic continuous functions on fractals, CHAOS SOLITONS & FRACTALS 45: (12) pp. 1579-1589., 2012.

[9] Z. Buczolich and J. Bremont: Maximizing points and coboundaries for an irrational rotation on the Circle, Ergodic Theory and Dynamical Systems, volume 33, issue 01, pp. 24-48., 2013.

[10] Buczolich Zoltán: Averages along the squares on the torus, Ergodic Theory and Dynamical Systems Proceedings of the Ergodic Theory Workshops at University of North Carolina at Chapel Hill, 2011-2012, 67-80, 2014.

[11] Z. Buczolich and S. Seuret: Measures and functions with prescribed homogeneous multifractal spectrum, Journal of Fractal Geometry, 1, No. 3, 295-333 (2014).

[12] R. Balka, Z. Buczolich and M. Elekes: A new fractal dimension: The topological Hausdorff dimension, Advances in Mathematics, to appear.

[13] Z. Buczolich and G. Keszthelyi: Convergence of ergodic averages for many group rotations, Ergodic Theory and Dynamical Systems, to appear.

[14] Z. Buczolich and S. Seuret: Homogeneous multifractal measures with disjoint spectrum and monohölder monotone functions, submitted, preprint available: http://www.cs.elte.hu/ buczo/papers/hmm2.pdf.

Attachment with MTMT data base information

Zoltán Buczolich's Selected Publications (OTKA elszámolási időszak publikációi)

2014

1 Buczolich Zoltán
Averages along the squares on the torus: Averages along the squares on the torus. Ergodic theory and dynamical systems, 67–79, De Gruyter Proc. Math., De Gruyter, Berlin, 2014.
In: Idris Assani (ed.)
Ergodic theory and dynamical systems,. Berlin: De Gruyter, 2014. pp. 67-79.
(ISBN:978-3-11-029813-0)
Chapter in Book/Chapter/Scientific

2013

2 Z Buczolich, J Bremont
Maximizing points and coboundaries for an irrational rotation on the Circle
ERGODIC THEORY AND DYNAMICAL SYSTEMS 33:(01) pp. 24-48. (2013)
Link(ek): DOI, Mathematical Reviews, WoS, Scopus, Egyéb URL
Journal Article/Article/Scientific

2012

3 Balka R, Buczolich Z, Elekes M
Topological Hausdorff dimension and level sets of generic continuous functions on fractals
CHAOS SOLITONS & FRACTALS 45:(12) pp. 1579-1589. (2012)
Link(ek): DOI, Mathematical Reviews, WoS, Scopus, arXiv
Journal Article/Article/Scientific
Independent citations: 4 Dependent (self-) citations: 2 All citations: 6

1. * *Balka R*
Measure and Category in Real Analysis
97 p. 2012.
Thesis/PhD/Scientific [12947138]

2 *Liu Jia, Wu Jun*
A remark on decomposition of continuous functions
Journal of Mathematical Analysis and Applications 401: (1) *pp. 404-406. (2013)*
Link(ek): DOI, Egyéb URL
Journal Article/Article/Scientific [13654863]

3 *Sun S, Li Z, Wu Y, Xu Y*
Contracting similarity fixed point of general Sierpinski gasket
Journal of Multimedia 8: (6) *pp. 816-822. (2013)*
Link(ek): DOI, Scopus
References
Journal Article [13519137]

4 *Bayart Frédéric, Heurteaux Yanick*
On the Hausdorff dimension of graphs of prevalent continuous functions on compact sets
In: Further Developments in Fractals and Related Fields. Springer, 2013. (ISBN 0817683992) *pp. 25-34.*
Chapter in Book [13618768]

5 * *Balka R*
Inductive topological Hausdorff dimensions and fibers of generic continuous functions
MONATSHEFTE FUR MATHEMATIK (ISSN: 0026-9255) 174: (1) *pp. 1-28. (2014)*
Link(ek): DOI, WoS, Scopus
Journal Article [14329695]

6 *G PANTSULAIA, M KINTSURASHVILI*
WHY IS THE NULL HYPOTHESIS REJECTED FOR "ALMOST EVERY" INFINITE SAMPLE BY SOME HYPOTHESIS TESTING OF MAXIMAL RELIABILITY?
Journal of Statistics: Advances in Theory and Applications 11: (1) *pp. 45-70. (2014)*
Journal Article/Article/Scientific [14089185]

2011

4 Z Buczolich, S Seuret
Singularity spectrum of generic αα-Hölder regular functions after time subordination.
JOURNAL OF FOURIER ANALYSIS AND APPLICATIONS 17:(3) pp. 457-485. (2011)
Link(ek): DOI, Mathematical Reviews, WoS, Scopus
Journal Article/Article/Scientific
Independent citations: 1 All citations: 1

1 *E KORCZAK-KUBIAK, A LORANTY, R J PAWLAK*
BAIRE GENERALIZED TOPOLOGICAL SPACES, GENERALIZED METRIC SPACES AND INFINITE GAMES: 140 (3) (2013), 203–231
ACTA MATHEMATICA ACADEMIAE SCIENTIARUM HUNGARICAE (ISSN: 0001-5954) 140: (3) *pp. 203-231. (2013)*
Journal Article [14344075]

5 Z Buczolich, S Seuret
Multifractal spectrum and generic properties of functions monotone in several variables.
JOURNAL OF MATHEMATICAL ANALYSIS AND APPLICATIONS 382:(1) pp. 110-126. (2011)
Link(ek): DOI, Mathematical Reviews, WoS, Scopus
Journal Article/Article/Scientific
Independent citations: 1 All citations: 1

1. *Moez Ben abid*
MULTIFRACTAL FORMALISM FOR TYPICAL PROBABILITY MEASURES ON SELF-SIMILAR SETS
http://arxiv.org/pdf/1205.6707v3. (2012.)
Miscellaneous/Publication in repository/Scientific [14330506]

2010

6 Assani, I, Buczolich Zoltán
The (Lp, Lq) bilinear Hardy-Littlewood function for the tail
ISRAEL JOURNAL OF MATHEMATICS 179:(1) pp. 173-187. (2010)
Link(ek): DOI, WoS, Scopus, Egyéb URL
Journal Article/Article/Scientific

7 Z Buczolich
Occupation measure and level sets of the Weierstrass-Cellerier function.
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