OTKA 075016 project closing report

Together with coauthors, we have investigated the mean width of the convex hull of n random points in a convex body with a rolling ball. The asymptotics of the mean width difference and the order of the variance were determined, and the strong law of large numbers is verified. These results were only known before if the convex body has twice differentiable boundary with positive curvature.

Let K be a d-dimensional convex body with a twice continuously differentiable boundary and everywhere positive Gauss-Kronecker curvature. Denote by K_n the convex hull of n points chosen randomly and independently from K according to the uniform distribution. Matching lower and upper bounds are obtained for the orders of magnitude of the variances of the sth intrinsic volumes $V_s(K_n)$ of K_n for $s \in \{1, , d\}$. Furthermore, strong laws of large numbers are proved for the intrinsic volumes of K_n. The essential tools are the economic cap covering theorem of Barany and Larman, and the Efron-Stein jackknife inequality.

Together with coauthors, we have proved a stability version of the classical Prékopa-Leindler inequality in the one dimensional case, and in the the case of even functions in higher dimensions. These results yield better error terms for the stability versions of the classical Blaschke-Santaló and affin isoperimetric inequalities.

The Petty projection inequality and its L_p extension by Lutwak, Yang and Zhang, and are fundamental affine isoperimetric and affine analytic inequalities. We verify a conjecture of Lutwak, Yang, Zhang about the equality case in the Orlicz-Petty projection inequality, and provide an essentially optimal stability version.

Let $\Xi_0 = [1, 1]$, and define the segments Ξ_n recursively in the following manner: for every $n = 0, 1, \ldots$, let $\Xi_{n+1} = \Xi_n \cap [a_{n+1} - 1, a_{n+1} + 1]$, where the point a_{n+1} is chosen randomly on the segment Ξ_n with uniform distribution. For the radius ϱ_n of Ξ_n, we prove that $n(\varrho_n - 1/2)$ converges in distribution to an exponential law, and we show that the centre of the limiting unit interval has arcsine distribution.

As a version of the classical polarization problem, it is proven that for any system of n points $z_1, \ldots, z_n$ on the (complex) unit circle, there exists another point z of norm 1, such that

$$\sum_{k=1}^{n} \frac{1}{|z_k - z|^2} \leq \frac{n^2}{4}.$$

Two proofs are presented: one uses a characterisation of equioscillating rational functions, while the other is based on Bernstein's inequality

The Helly dimension of a convex body K in R^d denoted by him(K) is the smallest integer k such that each family of translates of K intersects whenever any k+1 of them intersect. The L_1 sum of two convex bodies K_1 and K_2 such that the linear spaces $\mathrm{lin}(K_1)$ and $\mathrm{lin}(K_2)$ are independent is the polytope $\mathrm{conv}(K_1 \cup K_2)$. We have proved sharp lower bounds on the Helly dimension of the L_1 sum in terms of the Helly dimension of the constituents.

For an origin symmetric convex body in R^d of volume one, its cone volume measure is the probability measure on the sphere S^{d-1}, which is the integral of the normalized support function with respect to the surface area measure. Since a paper by Gromov, Milman from 1987, the cone volume measure has been discussed in various aspects. Under certain regularity conditions, finding the associated convex body for a given cone volume measure is equivalent to solve a Monge-Ampère type partial differential equation for the support function. Surface area measures of convex bodies on S^{d-1} were characterized by Minkowski in 1897. Together with coauthors, we characterized the cone volume measures among even probability measures on S^{d-1} in a paper at the prestigous Journal of AMS.

Unlike in the case of surface area measures, the associated origin symmetric convex body to a cone volume measure may not be unique. To describe uniqueness of the solution, we conjectured a logarithmic Brunn-Minkowski inequality, and verified the planar case.