Closing Report for OTKA Grant K60708 "Asymptotic Behaviour of Interacting Particle Systems: Fluctuations, Limit Distributions, Hydrodynamic Limits"

March 25, 2012

1 Summary of main results obtained during the support period

Interacting particle systems: constructions, stationarity, ergodicity, fluctuations, hydrodynamic limits

We proved existence of dynamics of infinitely extended systems of interacting particles of *zero-range* or *bricklayer* type, relaxing considerably the condition of linear growth (with the occupation number) of the interaction strength, [Balázs, Rassoul-Agha, Seppäläinen, Sethuraman (2007)].

We established precise relations between the fluctuation of current and the displacement of the second class particle in a very general class of 1-dimensional models of interacting particles with attractive interaction, [Balázs, Seppäläinen, Sethuraman (2007)]. Using these identities and various coupling arguments we contributed to rigorous mathematical foundation of Kardar-Parisi-Zhang theory of fluctuations in 1-dimensional systems of interacting particles like TASEP, ASEP, zero-range, bricklayers, etc. These results also led to scaling results on some solutions of the KPZ equation [Balázs, Komjáthy (2008)], [Balázs, Seppäläinen (2009)], [Balázs, Seppäläinen (2010)], [Balázs, Komjáthy, Seppäläinen (2012a)], [Balázs, Komjáthy, Seppäläinen (2012b)], [Balázs, Quastel, Seppäläinen (2011)].

We investigated the generalized solutions of Korteweg-de Vries equation with measure valued initial conditions. We proved that the white noise is generalized steady state of KdV evolution. [Quastel, Valkó (2010)], [Oh, Quastel, Valkó, Virág (2012)]. We determined the scaling exponent of a Brownian directed polymer model introduced by O'Connell and Yor [Seppäläinen, Valkó (2010)].

We investigated the conditions for imbedding finitely exchangeable measures into infinitely exchangeable ones, for systems arising in mean field models of statistical physics

[Liggett, Steif, Tóth (2007)]

Applying the resolvent method we proved super-diffusive estimates for fluctuations of various large stochastic systems. We proved that the finite-range exclusion processes in one dimension have the same order of diffusivity as the TASEP, [Quastel, Valkó (2007)], [Quastel, Valkó (2008)]. We showed the super-diffusivity of a family of one-dimensional self interacting polymers, [Tarrès, Tóth, Valkó (2012)]. We gave logarithmic super-diffusive bounds on the two-dimensional version of the model and also for certain two-dimensional random processes in random environment, [Tóth, Valkó (2012)].

Hyperbolic scaling limits of hyperbolic interacting particle systems were investigated. In contrast to diffusive models, the resulting Euler equations of hydrodynamics develop shocks in a finite time. That is why the derivation of the macroscopic equations from a microscopic model requires a synthesis of probabilistic and PDE methods. In the case of two-component stochastic models with a hyperbolic scaling law the method of compensated compactness seems to be the only tool that we can apply. Since the associated Lax entropies are not preserved by the microscopic dynamics, a logarithmic Sobolev inequality is needed to evaluate entropy production. Extending the arguments of [Shearer (1994)] and [Serre,Shearer (1994)] to stochastic systems, the nonlinear wave equation of isentropic elastodynamics is derived as the hyperbolic scaling limit of the anharmonic chain with Ginzburg-Landau type random perturbations. The study of interacting exclusion of charged particles with an additional creation and annihilation mechanism (electrophoresis) requires a new technique: the missing logarithmic Sobolev inequality is replaced by an associated relaxation scheme. In this case the uniqueness of the limit is also known. [Fritz, Nagy (2006)], [Fritz (2010a)], [Fritz (2010a)], [Bahadoran, Fritz, Nagy (2011)], [Fritz (2011)], [Fritz (2011)].

We investigated a one dimensional interacting system in which particles jump on $\mathbb{R}$, and the interaction is not local: it acts via the center of mass of the particles. We proved the fluid limit of the process as more and more particles are added, and investigated the limiting integro-differential equation, [Balázs, Rácz, Tóth (2012)].

We considered the problem of self-diffusion in deterministic mechanical systems, with random initial conditions. We proved particular limiting behaviour for the Rayleigh gas, in the light-mass limit. We established surprising relations with the Calogero-Moser model, [Bálint, Tóth, Tóth (2007)]

Random walks with long memory: random walks in random environments, self-interacting random walks, local time

We investigated the long time asymptotic behaviour of self-repelling random walks and diffusions, where the self-repellence of the trajectory is realized through a pushing force proportional to the gradient of the occupation time at the actual position. The models have their origin in both the physics and probability literature, under the names of *true self-avoiding random walk*, respectively, *self-repelling Brownian polymers*. For particular cases we proved limit theorems under time-to-the-two-thirds scaling in 1-dimension, [Tóth, Vető (2011)]. We found interesting identities and formulas for the expression of the density of the distribution of the so-called *true self-repelling motion*, [Dumaz, Tóth (2012)].

In general cases, using robust analytic methods, we proved superdiffusive bounds in one and two dimensions, [Tarrès, Tóth, Valkó (2012)], [Tarrès, Valkó (2012)]. In three and more dimensions we proved diffusive behaviour and central limit theorems, [Horváth, Tóth, Valkó (2012a)]. As a by-product we proved a new sufficient condition (a so-called sector condition) that guarantees central limit theorem for additive functionals of Markov processes that generalizes several already known sector conditions, [Horváth, Tóth, Valkó (2012b)].

We introduced a new class of self-interacting random walks in one-dimension, where the interactions are defined in terms of higher order derivatives (rather than gradient) of the occupation time measure. We identified non-rigorousely a rich phase diagram in terms of various interaction strength, [Erschler, Tóth, Werner (2012a)]. We proved part of these conjectures, establishing trapping in a particular range of the interaction parameters, [Erschler, Tóth, Werner (2012b)].

Random graphs and networks evolving in time

We established detailed local asymptotic behaviour for randomly growing networks with generalized (nonlinear) preferential attachment mechanism. We described in full detail the local structure of the emerging network, [Rudas, Tóth, Valkó (2007)], [Rudas, Tóth (2009)]. We also identified the Hausdorff dimension of the topological boundary of the emerging network, [Rudas, Tóth (2012)].

We investigated a coagulation-fragmentation model based on Erdős-Rényi random graphs, but with so-called forest fires added, which destroy the emerging giant cluster. We established the so-called self-organized criticality (SOC), i.e. the system sticks dynamically to a permanent critical state, [Ráth, Tóth (2009)]. Using similar methods we also established SOC for the mean-field version of Aldous's frozen percolation model, [Ráth, (2009)].

We extended the dense graph limit concept of Lovász and Szegedy to multigraphs and established relations to concepts of exchangeability, [Kolossváry, Ráth (2011)]. By use of these general concepts, we investigated the asymptotics of the edge-reconnecting preferential attachment model. As limiting description we obtained a cute stochastic differential equation, [Ráth, Szakács (2012)], [Ráth (2012)].

Random matrix theory

We derived the bulk scaling limit of the Gaussian beta-ensemble and we analyzed the large gap probabilities in the limit process, [Valkó, Virág (2009)], [Valkó, Virág (2010)]. We derived the bulk scaling limit of the Laguerre beta-ensemble, [Jaquot, Valkó (2011)]. We studied the spectrum of discrete random Schrodinger operators in one and two dimensions [Valkó, Virág (2011)], [Kritchevski, Valkó, Virág (2012)].

Mathematical finance

Our results obtained on modelling of financial defaults refer to the CDO (collateralized debt obligation) which is an insurance product. In the case of compound Poisson default process, we proved an analytic pricing formula by computing the Laplace transform of first passage times, and we demonstrated the applicability and the limits of the weighted Monte Carlo simulation, [Stippinger, Rácz, Veto, Bihary (2012)].

2 More detailed description of main results, based on abstracts of publications

Interacting particle systems 1: Constructions, stationarity, ergodicity

♠ **M. Balázs**, F. Rassoul-Agha, T. Seppäläinen, S. Sethuraman: Existence of the zero range process and a deposition model with superlinear growth rates, *Annals of Probability* **35:** 1201-1249 (2007)

We give a construction of the zero range and bricklayers' processes in the totally asymmetric, attractive case. The novelty is that we allow jump rates to grow exponentially. Earlier constructions have permitted at most linearly growing rates. We also show the invariance and extremality of a natural family of i.i.d. product measures indexed by particle density. Extremality is proved with an approach that is simpler than existing ergodicity proofs.

♠ **M. Balázs**, T. Seppäläinen: Exact connections between current fluctuations and the second class particle in a class of deposition models, *Journal of Statistical Physics*, **127:** 431-455 (2007)

We consider a large class of nearest neighbor attractive stochastic interacting systems that includes the asymmetric simple exclusion, zero range, bricklayers' and the symmetric K-exclusion processes. We provide exact formulas that connect particle flux (or surface growth) fluctuations to the two-point function of the process and to the motion of the second class particle. Such connections have only been available for simple exclusion where they were of great use in particle current fluctuation investigations.

♠ **M. Balázs**, T. Seppäläinen: A convexity property of expectations under exponential weights, *preprint/unpublished* (2007)

Take a random variable X with some finite exponential moments. Define an exponentially weighted expectation by $E^t(f) := E(e^{tX} f)/E(e^{tX})$ for admissible values of the parameter t. Denote the weighted expectation of X itself by $r(t) = E^t(X)$, with inverse function $t(r)$. We prove that for a convex function f the expectation $E^{t(r)}(f)$ is a convex function of the parameter r. Along the way we develop correlation inequalities for convex functions. Motivation for this result comes from equilibrium investigations of some stochastic interacting systems with stationary product distributions. In particular, convexity of the hydrodynamic flux function follows in some cases.

♠ **M. Balázs**, Gy. Farkas, P. Kovács, A. Rákos: Random walk of second class particles in product shock measures, *Journal of Statistical Physics*, **139:** 252-279 (2010)

We consider shock measures in a class of conserving stochastic particle systems on $\mathbb{Z}$. These shock measures have a product structure with a step-like density profile and include a second class particle at the shock position. We show for the asymmetric simple exclusion process, for the exponential bricklayers' process, and for a generalized zero range process, that under certain conditions these shocks, and therefore the second class particles, perform a simple random walk. Some previous results, including random walks of product shock measures and stationary shock measures seen from a second class particle, are direct consequences of our more general theorem. Multiple shocks can also be handled easily in this framework. Similar shock structure is also found in a non-conserving model, the branching coalescing random walk, where the role of the second class particle is played by the rightmost (or leftmost) particle.

♠ G. Amir, O. Angel, **B. Valkó**: The TASEP speed process, *Annals of Probability*, **39:** 1205-1242 (2011)

In a multi-type totally asymmetric simple exclusion process (TASEP) on the line, each site of $\mathbb{Z}$ is occupied by a particle labeled with a number and two neighboring particles are interchanged at rate one if their labels are in increasing order. Consider the process with the initial configuration where each particle is labeled by its position. It is known that in this case a.s. each particle has an asymptotic speed which is distributed uniformly on $[-1, 1]$. We study the joint distribution of these speeds: the TASEP speed process. We prove that the TASEP speed process is stationary with respect to the multi-type TASEP dynamics. Consequently, every ergodic stationary measure is given as a projection of the speed process measure. By relating this form to the known stationary measures for multi-type TASEPs with finitely many types we compute several marginals of the speed process, including the joint density of two and three consecutive speeds. One striking property of the distribution is that two speeds are equal with positive probability and for any given particle there are infinitely many others with the same speed. We also study the (partially) asymmetric simple exclusion process (ASEP). We prove that the ASEP with the above initial configuration has a certain symmetry. This allows us to extend some of our results, including the stationarity and description of all ergodic stationary measures, also to the ASEP.

♠ J. Quastel, **B. Valkó**: KdV preserves white noise, *Communicatoins in Mathematical Physics*, **277:** 707-714 (2008)

It is shown that white noise is an invariant measure for the Korteweg-deVries equation on $\mathbb{T}$. This is a consequence of recent results of Kappeler and Topalov establishing the well-posedness of the equation on appropriate negative Sobolev spaces, together with a result of Cambronero and McKean that white noise is the image under the Miura transform

(Ricatti map) of the (weighted) Gibbs measure for the modified KdV equation, proven to be invariant for that equation by Bourgain.

♠ T. Oh, J. Quastel, **B. Valkó**, B. Virág: Interpolation of Gibbs measures with white noise for Hamiltonian PDE, *Journal de Mathématiques Pures et Appliquées*, **97:** 391-410 (2012)

We consider the family of interpolation measures of Gibbs measures and white noise given by

$$dQ^{(p)}_{0,\beta} = Z_\beta^{-1} \mathbb{1}_{\{\int_{\mathbb{T}} u^2 \le K\beta^{-1/2}\}} e^{-\int_{\mathbb{T}} u^2 + \beta \int u^p} dP_{0,\beta}$$

where $P_{0,\beta}$ is the Wiener measure on the circle, with variance β^{-1}, conditioned to have mean zero. It is shown that as $\beta \to 0$, Q_0^β converges weakly to mean zero Gaussian white noise Q_0. As an application, we present a straightforward proof that Q_0 is invariant for the Kortweg-de Vries equation (KdV). This weak convergence also shows that the white noise is a weak limit of invariant measures for the modified KdV and the cubic nonlinear Schrödinger equations.

♠ T. M. Liggett, J. E. Steif, **B. Tóth**: Statistical mechanical systems on complete graphs, infinite exchangeability, finite extensions and a discrete moment problem, *Annals of Probability*, **35:** 867-914 (2007)

We show that a large collection of statistical mechanical systems with quadratically represented Hamiltonians on the complete graph can be extended to infinite exchangeable processes. This extends a known result for the ferromagnetic Curie–Weiss Ising model and includes as well all ferromagnetic Curie–Weiss Potts and Curie–Weiss Heisenberg models. By de Finetti's theorem, this is equivalent to showing that these probability measures can be expressed as averages of product measures. We provide examples showing that "ferromagnetism" is not however in itself sufficient and also study in some detail the Curie–Weiss Ising model with an additional 3-body interaction. Finally, we study the question of how much the antiferromagnetic Curie–Weiss Ising model can be extended. In this direction, we obtain sharp asymptotic results via a solution to a new moment problem. We also obtain a "formula" for the extension which is valid in many cases.

Interacting particle systems 2: Steady state and current fluctuations in non-reversible systems

♠ **J. Fritz**, **K. Nagy**, S. Olla: Equilibrium fluctuations for a system of harmonic oscillators with conservative noise, *Journal of Statistical Physics*, **122:** 399-415 (2006)

We investigate the harmonic chain forced by an additive noise, the evolution is given by an infinite system of stochastic differential equations. Total energy and deformation are preserved, the conservation of momentum is destroyed by the noise. Gaussian product measures are the extremal stationary states of this model. Equilibrium fluctuations of the conserved fields at a diffusive scaling are described by a couple of generalized Ornstein-Uhlenbeck processes.

♠ **M. Balázs**, E. Cator, T. Seppäläinen: Cube root fluctuations for the corner growth model associated to the exclusion process, *Electronic Journal of Probability*, **11:** 1094-1132 (2006)

We study the last-passage growth model on the planar integer lattice with exponential weights. With boundary conditions that represent the equilibrium exclusion process as seen from a particle right after its jump we prove that the variance of the last-passage time in a characteristic direction is of order $t^{2/3}$. With more general boundary conditions that include the rarefaction fan case we show that the last-passage time fluctuations are still of order $t^{1/3}$, and also that the transversal fluctuations of the maximal path have order $t^{2/3}$. We adapt and then build on a recent study of Hammersley's process by Cator and Groeneboom, and also utilize the competition interface introduced by Ferrari, Martin and Pimentel. The arguments are entirely probabilistic, and no use is made of the combinatorics of Young tableaux or methods of asymptotic analysis.

♠ **M. Balázs**, **J. Komjáthy**: Order of current variance and diffusivity in the rate one totally asymmetric zero range process, *Journal of Statistical Physics*, **133:** 59-78 (2008)

We prove that the variance of the current across a characteristic is of order $t^{2/3}$ in a stationary constant rate totally asymmetric zero range process, and that the diffusivity has order $t^{1/3}$. This is a step towards proving universality of this scaling behavior in the class of one-dimensional interacting systems with one conserved quantity and concave hydrodynamic flux. The proof proceeds via couplings to show the corresponding moment bounds for a second class particle. We build on the methods developed by Balazs-Seppäläinen for asymmetric simple exclusion. However, some modifications were needed to handle the larger state space. Our results translate into $t^{2/3}$-order of variance of the tagged particle on the characteristics of totally asymmetric simple exclusion.

♠ **M. Balázs**, T. Seppäläinen: Fluctuation bounds for the asymmetric simple exclusion process, *ALEA - Latin American Journal of Probability and Mathematical Statistics*, **6:** 1-24 (2009)

We give a partly new proof of the fluctuation bounds for the second class particle and current in the stationary asymmetric simple exclusion process. One novelty is a coupling that preserves the ordering of second class particles in two systems that are themselves ordered coordinatewise

♠ **M. Balázs**, T. Seppäläinen: Order of current variance and diffusivity in the asymmetric simple exclusion process, *Annals of Mathematics*, **171:** 1237-1265 (2010)

We prove that the variance of the current across a characteristic is of order $t^{2/3}$ in a stationary asymmetric simple exclusion process, and that the diffusivity has order $t^{1/3}$. The proof proceeds via couplings to show the corresponding results for the expected deviations and variance of a second class particle.

♠ **M. Balázs**, J. Quastel, T. Seppäläinen: Scaling exponent for the Hopf-Cole solution of KPZ/Stochastic Burgers, *Journal of the American Mathematical Society*, **24:** 683-708. (2011)

We consider the stochastic heat equation $\partial_t Z = \partial_x^2 Z - Z\dot{W}$ on the real line, where $\dot{W}$ is space-time white noise. $h(t,x) = -\log Z(t,x)$ is interpreted as a solution of the KPZ equation, and $u(t,x) = \partial_x h(t,x)$ as a solution of the stochastic Burgers equation. We take $Z(0,x) = \exp\{B(x)\}$ where $B(x)$ is a two-sided Brownian motion, corresponding to the stationary solution of the stochastic Burgers equation. We show that there exist $0 < c_1 \le c_2 < \infty$ such that $c_1 t^{2/3} \le \mathbf{Var}\big(\log Z(t,x)\big) \le c_2 t^{2/3}$. Analogous results are obtained for some moments of the correlation functions of $u(t,x)$. In particular, it is shown that the excess diffusivity satisfies $c_1 t^{1/3} \le D(t) \le c_2 t^{1/3}$. The proof uses approximation by weakly asymmetric simple exclusion processes, for which we obtain the microscopic analogies of the results by coupling.

♠ **M. Balázs**, **J. Komjáthy**, T. Seppäläinen: Microscopic concavity and fluctuation bounds in a class of deposition processes, *Annales de l'Institut Henri Poincaré - Probabilités et Statistiques*, **48:** 151-187. (2012)

We prove fluctuation bounds for the particle current in totally asymmetric zero range processes in one dimension with nondecreasing, concave jump rates whose slope decays exponentially. Fluctuations in the characteristic directions have order of magnitude $t^{1/3}$. This is in agreement with the expectation that these systems lie in the same KPZ universality class as the asymmetric simple exclusion process. The result is via a robust argument formulated for a broad class of deposition-type processes. Besides this class of zero range processes, hypotheses of this argument have also been verified in the authors' earlier papers for the asymmetric simple exclusion and the constant rate zero range processes, and are currently under development for a bricklayers process with exponentially increasing jump rates.

♠ **M. Balázs**, **J. Komjáthy**, T. Seppäläinen: Superdiffusivity and fluctuation bounds for the exponential bricklayers process, *Journal of Statistical Physics*, (2012, to appear)

This paper is the continuation of our earlier paper, where we proved $t^{1/3}$-order of current fluctuations across the characteristics in a class of one dimensional interacting systems with one conserved quantity. We also claimed two models with concave hydrodynamic flux which satisfied the assumptions which made our proof work. In the present note we show that the totally asymmetric exponential bricklayers process also satisfies these assumptions. Hence this is the first example with convex hydrodynamics of a model with $t^{1/3}$-order current fluctuations across the characteristics. As such, it further supports the idea of universality regarding this scaling.

♠ J. Quastel, **B. Valkó**: $t^{1/3}$ superdiffusivity of finite-range asymmetric exclusion processes on $\mathbb{Z}$, *Communications in Mathematical Physics* **273:** 379-394 (2007)

We consider finite-range asymmetric exclusion processes on $\mathbb{Z}$ with non-zero drift. The diffusivity $D(t)$ is expected to be of $\mathcal{O}(t^{1/3})$. We prove that $D(t) \ge Ct^{1/3}$ in the weak (Tauberian) sense that $\int_0^\infty e^{-\lambda t} t D(t) dt \ge C\lambda^{-7/3}$ as $\lambda \to 0$. The proof employs the resolvent method to make a direct comparison with the totally asymmetric simple exclusion process, for which the result is a consequence of the scaling limit for the two-point function recently obtained by Ferrari and Spohn. In the nearest neighbor case, we show further that $tD(t)$ is monotone, and hence we can conclude that $D(t) \geq Ct^{1/3}(\log t)^{-7/3}$ in the usual sense.

♠ J. Quastel, **B. Valkó**: A note on the diffusivity of finite-range asymmetric exclusion processes on $\mathbb{Z}$, In: V. Sidoravicius, M.E. Vares (eds): *In and Out of Equilibrium 2, Progress in Probability*, **60:** 543-550, Birkhäuser 2008

The diffusivity $D(t)$ of finite-range asymmetric exclusion processes on $\mathbb{Z}$ with non-zero drift is expected to be of order $t^{1/3}$. Seppäläinen and Balázs recently proved this conjecture for the nearest neighbor case. We extend their results to general finite range exclusion by proving that the Laplace transform of the diffusivity is of the conjectured order. We also obtain a pointwise upper bound for $D(t)$ the correct order.

♠ T. Säppälainen, **B. Valkó**: Bounds for scaling exponents for a $1+1$ dimensional directed polymer in a Brownian environment, *ALEA - Latin American Journal of Probability and Mathematical Statistics*, **10:** 451-476, (2010)

We study the scaling exponents of a $1+1$-dimensional directed polymer in a Brownian random environment introduced by O'Connell and Yor. For a version of the model with boundary conditions that are stationary in a space-time sense we identify the exact values of the exponents. For the version without the boundary conditions we get the conjectured upper bounds on the exponents.

♠ P. Bálint, **B. Tóth**, P. Tóth: On the zero mass limit of tagged particle diffusion in the 1-d Rayleigh gas, *Journal of Statistical Physics*, **127:** 657-675 (2007)

We consider the $M \to 0$ limit for tagged particle diffusion in a 1-dimensional Rayleigh-gas, studied originaly by Sinai and Soloveichik (1986), respectively, by Szasz and Toth (1986). In this limit we derive a new type of model for tagged paricle diffusion, with Calogero-Moser-Sutherland (i.e. inverse quadratic) interaction potential between the two central particles. Computer simulations on this new model reproduce exactly the numerical value of the limiting variance obtained by Boldrighini, Frigio and Tognetti (2002).

Interacting particle systems 3: Hydrodynamic limits

♠ **B. Valkó**: Hydrodynamic limit for perturbation of a hyperbolic equilibrium point in two-component systems, *Annales de l'Institut Henri Poincaré – Probabilités et Statisttiques*, **42:** 61-80 (2006)

We consider one-dimensional, locally finite interacting particle systems with two conservation laws. The models have a family of stationary measures with product structure and we assume the existence of a uniform bound on the inverse of the spectral gap which is quadratic in the size of the system. Under Eulerian scaling the hydrodynamic limit for the macroscopic density profiles leads to a two-component system of conservation laws.

The resulting pde is hyperbolic inside the physical domain of the macroscopic densities, with possible loss of hyperbolicity at the boundary. We investigate the propagation of small perturbations around a *hyperbolic* equilibrium point. We prove that the perturbations essentially evolve according to two *decoupled* Burgers equations. The scaling is not Eulerian: if the lattice constant is n^{-1}, the perturbations are of order $n^{-\beta}$ then time is speeded up by $n^{1+\beta}$. Our derivation holds for $0 < \beta < \frac{1}{5}$. The proof relies on Yau's relative entropy method, thus it applies only in the regime of smooth solutions.

♠ **J. Fritz**, **K. Nagy**: On uniqueness of the Euler limit of one-component lattice gas models, *ALEA Latin American Journal of Probability and Mathematical Statistics*, **1:** 367-392 (2006)

We investigate the interaction of one-dimensional asymmetric exclusion processes of opposite speeds, the exchange mechanism is combined with a spin-flip dynamics, and this asymmetric law is regularized by a nearest neighbor stirring of large intensity. At an intuitive level we can say that particles with ± 1 spins are subject to an external magnetic field, and the additional spin-flip dynamics results in a strong relaxation of total magnetization. Therefore this modification of the model of Fritz and Tóth (2004) admits particle number as the only conservation law, with hyperbolic scaling. By means of a two-step version of LSI based estimation techniques we prove that compensated compactness and the Lax entropy inequality imply the existence and uniqueness of the hydrodynamic limit even in a regime of shocks.

♠ **J. Fritz**: Hyperbolic scaling limits: The method of compensated compactness, *Markov Processes Related Fields*, **16:** 117-138 (2010)

♠ **J. Fritz**: Application of relaxation schemes in the microscopic theory of hydrodynamics, *Moscow Mathematical Journal*, **10:** 729–745 (2010)

♠ Ch. Bahadoran, **J. Fritz**, **K. Nagy**: Relaxation scheme for interacting exclusions, *Electronic Journal of Probability*, **16:** 230–262 (2011)

We investigate the interaction of one-dimensional asymmetric exclusion processes of opposite speeds, where the exchange dynamics is combined with a creation-annihilation mechanism, and this asymmetric law is regularized by a nearest neighbor stirring of large intensity. The model admits hyperbolic (Euler) scaling, and we are interested in the hydrodynamic behavior of the system in a regime of shocks on the infinite line. This work is a continuation of a previous paper by Fritz and Nagy 2006, where this question has been left open because of the lack of a suitable logarithmic Sobolev inequality. The problem is solved by extending the method of relaxation schemes to this stochastic model, the resulting a priory bound allows us to verify compensated compactness.

♠ **J. Fritz**: Microscopic theory of isothermal elastodynamics, *Archive for Rational Mechanics and Analysis*, **201:** 209-249 (2011)

This paper examines random perturbations of the anharmonic chain of coupled oscillators. The microscopic system has two conservation laws, and its hyperbolic scaling limit results in the quasi-linear wave equation (p-system) of isothermal (isentropic) elasticity. In the shock regime, the compensated compactness method is used. Lastly results from J. W. Shearer and D. Serre are applied.

♠ **J. Fritz**: Compensated compactness and relaxation schemes at the microscopic level, In: I. Fazekas (ed.): *Proceedings of the Conference Dedicated to Mátyás Arató on the Occasion of his 80th Birthday*, 2012 (to appear)

This is a survey of some recent results on hyperbolic scaling limits. The resulting Euler equations of hydrodynamics develop shocks in a finite time, in such situations the derivation of the macroscopic equations from a microscopic model requires a synthesis of probabilistic and PDE methods. In the case of two-component stochastic models with a hyperbolic scaling law the method of compensated compactness seems to be the only tool that we can apply. Since the associated Lax entropies are not preserved by the microscopic dynamics, a logarithmic Sobolev inequality is needed to evaluate entropy production. Extending the arguments of Shearer (1994), respectively Serre and Shearer (1994) to stochastic systems, the nonlinear wave equation of isentropic elastodynamics is derived as the hyperbolic scaling limit of the anharmonic chain with Ginzburg-Landau type random perturbations. The model of interacting exclusion of charged particles results in the Leroux system in a similar way. In the presence of an additional creation/annihilation mechanism the missing logarithmic Sobolev inequality is replaced by an associated relaxation scheme. In this case the uniqueness of the limit is also known.

♠ **M. Balázs**, M. Z. Rácz, **B. Tóth**: Modeling flocks and prices: jumping particles with an attractive interaction, *preprint, submitted for publication* (2012)

We introduce and investigate a new model of a finite number of particles jumping forward on the real line. The jump lengths are independent of everything, but the jump rate of each particle depends on the relative position of the particle compared to the center of mass of the system. The rates are higher for those left behind, and lower for those ahead of the center of mass, providing an attractive interaction keeping the particles together. We prove that in the fluid limit, as the number of particles goes to infinity, the evolution of the system is described by a mean field equation that exhibits traveling wave solutions. A connection to extreme value statistics is also provided.

Random walks with long memory: rw in random environment, self-interacting rw, local time

♠ **M. Balázs**, F. Rassoul-Agha, T. Seppäläinen: The random average process and random walk in a space-time random environment in one dimension, *Communications in Mathematical Physics*, **266:** 499-545 (2006)

We study space-time fluctuations around a characteristic line for a one-dimensional interacting system known as the random average process. The state of this system is a real-valued function on the integers. New values of the function are created by averaging previous values with random weights. The fluctuations analyzed occur on the scale $n^{1/4}$ where n is the ratio of macroscopic and microscopic scales in the system. The limits of the fluctuations are described by a family of Gaussian processes. In cases of known product-form equilibria, this limit is a two-parameter process whose time marginals are fractional Brownian motions with Hurst parameter 1/4. Along the way we study the limits of quenched mean processes for a random walk in a space-time random environment. These limits also happen at scale $n^{1/4}$ and are described by certain Gaussian processes that we identify. In particular, when we look at a backward quenched mean process, the limit process is the solution of a stochastic heat equation.

♠ **B. Tóth**, **B. Vető**: Skorohod-reflection of Brownian paths and BES^3, *Acta Sci. Math. (Szeged)* **73:** 781-788 (2007)

Let $B(t)$, $X(t)$ and $Y(t)$ be independent standard $1d$ Borwnian motions. Define $X^+(t)$ and $Y^-(t)$ as the trajectories of the processes $X(t)$ and $Y(t)$ pushed upwards and, respectively, downwards by $B(t)$, according to Skorohod-reflection. In a recent paper, Jon Warren proves inter alia that $Z(t) := X^+(t) - Y^-(t)$ is a three-dimensional Bessel-process. In this note, we present an alternative, elementary proof of this fact.

♠ **B. Tóth**, **B. Vető**: Self repelling random walk with directed edges on $\mathbb{Z}$, *Electronic Journal of Probability*, **13:** 1909-1926 (2008)

We consider a variant of self-repelling random walk on the integer lattice $\mathbb{Z}$ where the self-repellence is defined in terms of the local time on oriented edges. The long-time asymptotic scaling of this walk is surprisingly different from the asymptotics of the similar process with self-repellence defined in terms of local time on unoriented edges. We prove limit theorems for the local time process and for the position of the random walker. The main ingredient is a Ray-Knight-type of approach. At the end of the paper, we also present some computer simulations which show the strange scaling behaviour of the walk considered.

♠ **B. Tóth**, **B. Vető**: Continuous time "true self-avoiding" random walk on Z, *ALEA - Latin American Journal of Probability and Mathematical Statistics* **8:** 59-75 (2011)

We consider the continuous time version of the 'true' or 'myopic' selfavoiding random walk with site repulsion in 1d. The Ray-Knight-type method which was applied in (Tóth, 1995) to the discrete time and edge repulsion case is applicable to this model with some modifications. We present a limit theorem for the local time of the walk and a local limit theorem for the displacement.

♠ S. Athreya, S. Sethuraman, **B. Tóth**: On the range, local times and periodicity of random walk on an interval, *ALEA – Latin American Journal of Probability and Mathematical Statistics*, **8:** 269–284 (2011)

The range, local times, and periodicity of symmetric, weakly asymmetric and asymmetric random walks at the time of exit from a strip with N locations are considered. Several results on asymptotic distributions are obtained.

♠ A. Erschler, **B. Tóth**, W. Werner: Some locally self-interacting walks on the integers, In: J.-D. Deuschel, B Gentz, W König, M von Renesse, M Scheutzow, U Schmock (eds.): *Festschrift for Erwin Bolthausen and Jürgen Gärtner*, *Springer Proceedings in Mathematics*, Springer 2012 (to appear)

We study certain self-interacting walks on the set of integers, that choose to jump to the right or to the left randomly but influenced by the number of times they have previously jumped along the edges in the finite neighbourhood of their current position (in the present paper, typically, we will discuss the case where one considers the neighbouring edges and the next-to-neighbouring edges). We survey a variety of possible behaviours, including some where the walk is eventually confined to an interval of large length. We also focus on certain "asymmetric" drifts, where we prove that with positive probability, the walks behave deterministically on large scale and move like a constant times the square root of time, or like a constant times the logarithm of time.

♠ A. Erschler, **B. Tóth**, W. Werner: Stuck walks, *Probability Theory and Related Fields*, (2012, to appear) DOI 10.1007/s00440-011-0365-4

We investigate the asymptotic behaviour of a class of self-interacting nearest neighbour random walks on the one-dimensional integer lattice which are pushed by a particular linear combination of their own local time on edges in the neighbourhood of their current position. We prove that in a range of the relevant parameter of the model such random walkers can be eventually confined to a finite interval of length depending on the parameter value. The phenomenon arises as a result of competing self-attracting and self-repelling effects where in the named parameter range the former wins.

♠ P. Tarrès, **B. Tóth**, **B. Valkó**: Diffusivity bounds for $1d$ Brownian polymers, *Annals of Probability*, (2012, to appear)

We study the asymptotic behavior of a self interacting one dimensional Brownian polymer first introduced by Durrett and Rogers. The polymer describes a stochastic process with a drift which is a certain average of its local time. We show that a smeared out version of the local time function as viewed from the actual position of the process is a Markov process in a suitably chosen function space, and that this process has a Gaussian stationary measure. As a first consequence this enables us to partially prove a conjecture about the law of large numbers for the end-to-end displacement of the polymer formulated by Durrett and Rogers. Next we give upper and lower bounds for the variance of the process under the stationary measure, in terms of the qualitative infrared behavior of the interaction function. In particular we show that in the locally self-repelling case (when the process is essentially pushed by the negative gradient of its own local time) the process is super-diffusive.

♠ **I. Horváth**, **B. Tóth**, **B. Vető**: Diffusive limits for “true” (or myopic) self-avoiding random walks and self-repellent Brownian polymers in $d \geq 3$, *Probability Theory and Related Fields* (2012, to appear) DOI 10.1007/s00440-011-0358-3

The problems considered in the present paper have their roots in two different cultures. The ’true’ (or myopic) self-avoiding walk model (TSAW) was introduced in the physics literature by Amit, Parisi and Peliti. This is a nearest neighbor non-Markovian random walk in $\mathbb{Z}^d$ which prefers to jump to those neighbors which were less visited in the past. The self-repelling Brownian polymer model (SRBP), initiated in the probabilistic literature by Durrett and Rogers (independently of the physics community), is the continuous space-time counterpart: a diffusion in $\mathbb{R}^d$ pushed by the negative gradient of the (mollified) occupation time measure of the process. In both cases, similar long memory effects are caused by a pathwise self-repellency of the trajectories due to a push by the negative gradient of (softened) local time. We investigate the asymptotic behaviour of TSAW and SRBP in the non-recurrent dimensions. First, we identify a natural stationary (in time) and ergodic distribution of the environment (the local time profile) as seen from the moving particle. The main results are diffusive limits. In the case of TSAW, for a wide class of self-interaction functions, we establish diffusive lower and upper bounds for the displacement and for a particular, more restricted class of interactions, we prove full CLT for the finite dimensional distributions of the displacement. In the case of SRBP, we prove full CLT without restrictions on the interaction functions. These results settle part of the conjectures, based on non-rigorous renormalization group arguments (equally 'valid' for the TSAW and SRBP cases). The proof of the CLT follows the non-reversible version of Kipnis-Varadhan theory. On the way to the proof, we slightly weaken the so-called graded sector condition

♠ **I. Horváth**, **B. Tóth**, **B. Vető**: Relaxed sector condition, *preprint, submitted for publication* (2012)

In this note we present a new sufficient condition which guarantees martingale approximation and central limit theorem a la Kipnis-Varadhan to hold for additive functionals of Markov processes. This new condition, dubbed the relaxed sector condition generalizes several other well-known sector conditions like the (strong) sector condition and the graded sector condition, while also being interesting in its own right.

♠ **B. Tóth**, **B. Valkó**: Superdiffusive bounds on self-repellent Brownian polymers and diffusion in the curl of the Gaussian free field in $d = 2$, *Journal of Statistical Physics* (2012, to appear) DOI 10.1007/s10955-012-0462-5

We consider two models of random diffusion in random environment in two dimensions. The first example is the self-repelling Brownian polymer, this describes a diffusion pushed by the negative gradient of its own occupation time measure (local time). The second example is a diffusion in a fixed random environment given by the curl of massless Gaussian free field. In both cases we show that the process is superdiffusive: the variance grows faster than linearly with time. We give lower and upper bounds of the order of $t \log \log t$, respectively, $t \log t$. We also present computations for an anisotropic version of the self-repelling Brownian polymer where we give lower and upper bounds of $t(\log t)^{1/2}$, respectively, $t \log t$. The bounds are given in the sense of Laplace transforms, the proofs rely on the resolvent method. The true order of the variance for these processes is expected to be $t(\log t)^{1/2}$ for the isotropic and $t(\log t)^{2/3}$ for the non-isotropic case. In the appendix we present a non-rigorous derivation of these scaling exponents.

♠ L. Dumaz, **B. Tóth**: Marginal densities of the "true" self-repelling motion, *prerpint, submitted for publication* (2012)

Let $X(t)$ be the true self-repelling motion (TSRM) constructed in [Tóth, Werner (1998)], $L(t, x)$ its occupation time density (local time) and $H(t) := L(t, X(t))$ the height of the local time profile at the actual position of the motion. The joint distribution of $(X(t), H(t))$ was identified in [Tóth (1995)] in somewhat implicit terms. Now we give explicit formulas for the densities of the marginal distributions of $X(t)$ and $H(t)$. The distribution of $X(t)$ has a particularly surprising shape: It has a sharp local minimum with discontinuous derivative at 0. As a consequence we also obtain a precise version of the large deviation estimate of arXiv:1105.2948v2.

♠ **B. Vető**: *Asymptotic Behaviour of Random Walks with Long Memory*, PhD thesis, BME (2011)

Based on [Tóth, Vető (2007)], [Tóth, Vető (2008)], [Tóth, Vető (2011)], [Horváth, Tóth, Vető (2012a)] and [Horváth, Tóth, Vető (2012b)]. Defended with qualification *summa cum laude*, on June 28, 2011.

Random graphs, randomly growing networks, random combinatorial structures

♠ **A. Rudas**, **B. Tóth**, **B. Valkó**: Random trees and general branching processes, *Random Structures and Algorithms*, **31:** 186-202 (2007)

We consider a model of random tree growth, where at each time unit a new vertex is added and attached to an already existing vertex chosen at random. The probability with which a vertex with degree k is chosen is proportional to $w(k)$, where the weight function w is the parameter of the model. In the papers of B. Bollobas, O. Riordan, J. Spencer, G. Tusnady, and, independently, Mori, the asymptotic degree distribution is obtained for a model that is equivalent to the special case of ours, when the weight function is linear. The proof therein strongly relies on the linear choice of w. We give the asymptotical degree distribution for a wide range of weight functions. Moreover, we provide the asymptotic distribution of the tree itself as seen from a randomly selected vertex. The latter approach gives greater insight to the limiting structure of the tree. Our proof relies on the fact that considering the evolution of the random tree in continuous time, the process may be viewed as a general branching process, this way classical results can be applied.

♠ **A. Rudas**, **B. Tóth**: Random tree growth with branching processes – a survey, In: B. Bollobas, R. Kozma, D. Miklos (eds): *Handbook of Large-Scale Random Networks*; *Bolyai Society Math. Studies*, **18** Springer, Berlin-Heidelberg-New York, 2009

We investigate the asymptotic properties of a random tree growth model which generalizes the basic concept of preferential attachment. The Barabasi-Albert random graph model is based on the idea that the popularity of a vertex in the graph (the probability that a new vertex will be attached to it) is proportional to its current degree. The dependency on the degree, the so-called weight function, is linear in this model. We give results which are valid for a much wider class of weight functions. This generalized model has been introduced by Krapivsky and Redner in the physics literature. The method of rephrasing the model in a continuous-time setting makes it possible to connect the problem to the well-developed theory of branching processes. We give local results, concerning the neighborhood of a "typical" vertex in the tree, and also global ones, about the repartition of mass between subtrees under fixed vertices.

♠ **A. Rudas**, I.P. Tóth: Entropy and Hausdorff dimension in random growing trees, *Stochastics and Dynamics*, (2012, to appear)

We investigate the limiting behavior of random tree growth in preferential attachment models. The tree stems from a root, and we add vertices to the system one-by-one at random, according to a rule which depends on the degree distribution of the already existing tree. The so-called weight function, in terms of which the rule of attachment is formulated, is such that each vertex in the tree can have at most K children. We define the concept of a certain random measure mu on the leaves of the limiting tree, which captures a global property of the tree growth in a natural way. We prove that the Hausdorff and the packing dimension of this limiting measure is equal and constant with probability one. Moreover, the local dimension of mu equals the Hausdorff dimension at mu-almost every point. We give an explicit formula for the dimension, given the rule of attachment.

♠ **B. Ráth**, **B. Tóth**: Triangle percolation in mean field random graphs - with PDE, *Journal of Statistical Physics*, **131:** 385-391 (2008)

We apply a PDE-based method to deduce the critical time and the size of the giant component of the "triangle percolation" on the Erdős-Rényi random graph process investigated by Palla, Derényi and Vicsek

♠ **B. Ráth**, **B. Tóth**: Erdos-Renyi ramdom graphs + forest fires = self-organized criticality, *Electronic Journal of Probability*, **14:** 1290-1327 (2009)

We modify the usual Erdős-Rényi random graph evolution by letting connected clusters 'burn down' (i.e. fall apart to disconnected single sites) due to a Poisson flow of lightnings. In a range of the intensity of rate of lightnings the system sticks to a permanent.

♠ **B. Ráth**: Mean field frozen percolation, *Journal of Statistical Physics*, **137:** 459-499 (2009) We define a modification of the Erdos-Renyi random graph process which can be regarded as the mean field frozen percolation process. We describe the behavior of the process using differential equations and investigate their solutions in order to show the self-organized critical and extremum properties of the critical frozen percolation model. We prove two limit theorems about the distribution of the size of the component of a typical frozen vertex.

♠ I. Kolossváry, **B. Ráth**: Multigraph limits and exchangeability, *Acta Mathematica Hungarica*, **130:** 1-34 (2011)

We define the edge reconnecting model, a random multigraph evolving in time. At each time step we change one endpoint of a uniformly chosen edge: the new endpoint is chosen by linear preferential attachment. We consider a sequence of edge reconnecting models where the sequence of initial multigraphs is convergent in a sense which is a natural generalization of the notion of convergence of dense graph sequences. We investigate how the limit object evolves under the edge reconnecting dynamics if we rescale time properly: we give the complete characterization of the time evolution of the limit object from its initial state up to the stationary state using the theory of exchangeable arrays, queuing and diffusion processes. The number of parallel edges and the degrees evolve on different timescales and because of this the model exhibits subaging.

♠ **B. Ráth**, L. Szakács: Multigraph limit of the dense configuration model and the preferential attachment graph, *Acta Mathematica Hungarica*, (2012, to appear)

The configuration model is the most natural model to generate a random multigraph with a given degree sequence. We use the notion of dense graph limits to characterize the special form of limit objects of convergent sequences of configuration models. We apply these results to calculate the limit object corresponding to the dense preferential attachment graph and the edge reconnecting model. Our main tools in doing so are (1) the relation between the theory of graph limits and that of partially exchangeable random arrays (2) an explicit construction of our random graphs that uses urn models.

♠ **B. Ráth**: Time evolution of dense multigraph limits under edge-conservative preferential attachment dynamics, *Random Structures and Algorithms*, (2012, to appear)

We define the edge reconnecting model, a random multigraph evolving in time. At each time step we change one endpoint of a uniformly chosen edge: the new endpoint is chosen by linear preferential attachment. We consider a sequence of edge reconnecting models where the sequence of initial multigraphs is convergent in a sense which is a natural generalization of the notion of convergence of dense graph sequences. We investigate how the limit object evolves under the edge reconnecting dynamics if we rescale time properly: we give the complete characterization of the time evolution of the limit object from its initial state up to the stationary state using the theory of exchangeable arrays, queuing and diffusion processes. The number of parallel edges and the degrees evolve on different timescales and because of this the model exhibits subaging.

♠ C. Hoppen, Y. Kohayakawa, C. G. Moreira, **B. Ráth**, R. M. Sampaio: Limits of permutation sequences, Submitted for publication: *preprint, submitted for publication* (2011)

A permutation sequence is said to be convergent if the density of occurrences of every fixed permutation in the elements of the sequence converges. We prove that such a convergent sequence has a natural limit object, namely a Lebesgue measurable function $Z : [0,1]^2 \to [0,1]$ with the additional properties that, for every fixed $x \in [0,1]$, the restriction $Z(x,\cdot)$ is a cumulative distribution function and, for every $y \in [0,1]$, the restriction $Z(\cdot,y)$ satisfies a "mass" condition. This limit process is well-behaved: every function in the class of limit objects is a limit of some permutation sequence, and two of these functions are limits of the same sequence if and only if they are equal almost everywhere. An ingredient in the proofs is a new model of random permutations, which generalizes previous models and might be interesting for its own sake.

♠ **B. Ráth**: *Asymptotic Behavior of Random Graphs Evolving in Time*, PhD Thesis, BME (2010)

Based on [Ráth, Tóth (2008)], [Ráth, Tóth (2009)], [Ráth (2009)], [Kolossváry, Ráth (2011)], [Ráth (2012)], and [Ráth, Szakács (2012)]. Defended with qualification *summa cum laude*, on April 27, 2010.

Random matrix theory

♠ **B. Valkó**, B. Virág: Continuum limits of random matrices and the Brownian carousel, *Inventiones Mathematicae*, **177:** 463-508 (2009)

We show that at any location away from the spectral edge, the eigenvalues of the Gaussian unitary ensemble and its general beta-siblings converge to Sine-beta, a translation invariant point process. This process has a geometric description in term of the Brownian carousel, a deterministic function of Brownian motion in the hyperbolic plane. The Brownian carousel, a description of the a continuum limit of random matrices, provides a convenient way to analyze the limiting point processes. We show that the gap probability of Sine-beta is continuous in the gap size and β, and compute its asymptotics for large gaps. Moreover, the stochastic differential equation version of the Brownian carousel exhibits a phase transition at $beta = 2$.

♠ **B. Valkó**, B. Virág: Large gaps between random eigenvalues, *Annals of Probability*, **38:** 1263-1279 (2010)

We show that in the point process limit of the bulk eigenvalues of β-ensembles of random matrices, the probability of having no eigenvalue in a fixed interval of size λ is given by

$$\big(\kappa_\beta + o(1)\big)\lambda^{\gamma_\beta} \exp\left(-\frac{\beta}{64}\lambda^2 + \left(\frac{\beta}{8} - \frac{1}{4}\right)\lambda\right)$$

as $\lambda \to \infty$, where

$$\gamma_\beta = \frac{1}{4}\left(\frac{\beta}{2} + \frac{2}{\beta} - 3\right)$$

and κ_β is an undetermined positive constant. This is a slightly corrected version of a prediction by Dyson [*J. Math. Phys.* **3:** 157–165 (1962)]. Our proof uses the new Brownian carousel representation of the limit process, as well as the Cameron–Martin–Girsanov transformation in stochastic calculus.

♠ **B. Valkó**, B. Virág: Random Schrödinger operators on long boxes, noise explosion and the GOE, *preprint, submitted for publication* (2011)

It has been conjectured that the eigenvalues of random Schrodinger operators at the localization transition in dimensions $d \geq 2$ behave like the eigenvalues of the Gaussian Orthogonal Ensemble (GOE). We study a the eigenvalues of long boxes in dimension d=2 for low disorder. We deduce a stochastic differential equation representation for the limiting process. We show that in dimension $d = 2$ there are sequences of boxes so that the eigenvalues in low disorder converge to Sine1, the limiting eigenvalue process of the GOE.

♠ S. Jacquot, **B. Valkó**: Bulk scaling limit of the Laguerre ensemble, *Electronic Journal of Probability*, **16:** 314-346 (2011)

We consider the beta-Laguerre ensemble, a family of distributions generalizing the joint eigenvalue distribution of the Wishart random matrices. We show that the bulk scaling limit of these ensembles exists for all $\beta > 0$ for a general family of parameters and it is the same as the bulk scaling limit of the corresponding beta-Hermite ensemble.

♠ E. Kritchevski, **B. Valkó**, B. Virág: The scaling limit of the critical one-dimensional random Schrödinger operator, *Communications in Mathematical Physics*, (2012, to appear)

We consider two models of one-dimensional discrete random Schrodinger operators $(H_n\psi)_l = \psi_{l-1} + \psi_{l+1} + v_l\psi_l$, $\psi_0 = \psi_{n+1} = 0$ in the cases $v_k = \sigma\omega_k/\sqrt{n}$ and $v_k = \sigma\omega_k/\sqrt{k}$. Here ω_k are independent random variables with mean 0 and variance 1. We show that the eigenvectors are delocalized and the transfer matrix evolution has a scaling limit given by a stochastic differential equation. In both cases, eigenvalues near a fixed bulk energy E have a point process limit. We give bounds on the eigenvalue repulsion, large gap probability, identify the limiting intensity and provide a central limit theorem. In the second model, the limiting processes are the same as the point processes obtained as the bulk scaling limits of the beta-ensembles of random matrix theory. In the first model, the eigenvalue repulsion is much stronger.

Mathematics of finance

♠ B. Tóth, **B. Tóth**, J. Kertész: Modeling the Epps effect of crosscorrelations in asset prices, In: J. Kertész, R. Mantegna, S. Bornholdt (editors): *Fluctuations and Noise in Finance and Complex Systems, Florence 2007, Proceedings of SPIE Vol. 6601*, 2007

We review the decomposition method of stock return cross-correlations, presented previously for studying the dependence of the correlation coefficient on the resolution of data (Epps effect). Through a toy model of random walk/Brownian motion and memoryless renewal process (i.e. Poisson point process) of observation times we show that in case of analytical treatability, by decomposing the correlations we get the exact result for the frequency dependence. We also demonstrate that our approach produces reasonable fitting of the dependence of correlations on the data resolution in case of empirical data. Our results indicate that the Epps phenomenon is a product of the finite time decay of lagged correlations of high resolution data, which does not scale with activity. The characteristic time is due to a human time scale, the time needed to react to news.

♠ M. Stippinger, E. Rácz, **B. Vető**, Zs. Bihary: Analytic results and weighted Monte Carlo simulations for CDO pricing, *European Physical Journal B* (2012, to appear)

We explore the possibilities of importance sampling in the Monte Carlo pricing of a structured credit derivative referred to as Collateralized Debt Obligation (CDO). Modeling a CDO contract is challenging, since it depends on a pool of (typically ~ 100) assets, Monte Carlo simulations are often the only feasible approach to pricing. Variance reduction techniques are therefore of great importance. This paper presents an exact analytic solution using Laplace-transform and MC importance sampling results for an easily tractable intensity-based model of the CDO, namely the compound Poissonian. Furthermore analytic formulae are derived for the reweighting efficiency. The computational gain is appealing, nevertheless, even in this basic scheme, a phase transition can be found, rendering some parameter regimes out of reach. A model-independent transform approach is also presented for CDO pricing.

3 Other activities related to this grant

Selection of invited lectures

During the support period participants of this research project were regular invited speakers at prestigious international conferences, workshops, colloquia. A rough estimate gives a total of cca. 60 invited conference/workshop lectures and cca. 45 invited colloquium/seminar talks. Here follows a *very restricted selection of invited lectures* at conferences presented by the participants in this OTKA grant, in the last period (from end of 2008) of the support. A full list would be much too long for this summary report. For more exhaustive information in this respect see the personal home pages of the participants.

- *Interacting Particle Systems and Percolation*, Institut Henri Poncaré (IHP), Paris, October 2008 (**Balázs, Fritz, Tóth**)
- *Interacting stochastic particle systems*, Centre de Recherche Mathématiques (CRM), Montréal, May 2009, (**Balázs, Tóth, Valkó**)
- *27th European Meeting of Statisticians*, Toulouse, July 2009 (**Vető**)
- *International Congress on Mathematical Physics*, Prague, August 2009, (**Valkó**)
- *Large Scale Dynamics of Interacting Particle Systems*, Fukuoka, August 2009, (**Fritz**)
- *Seminar on Stochastic Processes 2010*, Orlando FL, March 2010 (**Valkó**)

- *Rencontres de Probabilités*, Rouen, May 2010, (**Balázs** (minicourse))
- *28th European Meeting of Statisticians*, Piraeus, July 2010 (**Valkó**)
- *34th Conference on Stochastic Processes and their Applications*, Osaka, August 2010 (**Tóth**)
- *Large Scale Dynamics of Interacting Particle Systems*, Tokyo, September 2010, (**Tóth**)
- *Large Scale Stochastic Dynamics*, Oberwolfach, November 2010 (**Balázs, Fritz, Tóth**)
- *Random Matrix Theory and its Applications II*, MSRI Berkeley, December 2010 (**Valkó**)
- *Probability Theory, Statistical Physics and Applications*, NYU Abu Dhabi, January 2011 (**Tóth**)
- *Easter Probability Meeting, Random Structures and Dynamics*, Oxford, April 2011 (**Ráth, Tóth**)
- *Entropy and Convexity for Nonlinear Partial Differential Equations*, Kavli Royal Society International Centre, June 2011 (**Fritz**)
- *35th Conference on Stochastic Processes and their Applications*, Oaxaca, July 2011 (**Valkó**)
- *Large Scale Dynamics of Interacting Particle Systems*, Kochi, December 2011, (**Fritz**)
- *Forest Fires and Self-Organized Criticality*, Toulouse, March 2012, (**Ráth, Tóth**)

Conferences organized

We organized the following international conferences and workshops

- *Randomness and Hyperbolicity in Dynamical Systems*, Budapest, 21-25 August 2006
- *Hydrodynamics and Fluctuations in Interacting Particle Systems*, Budapest, 27-29 March 2008
- *BME-Technion Joint Probability Workshop*, Haifa, 20-22 January 2009

Members of the team took part in the work of the scientific programme committees of *three SPA Conferences* (2006 Urbana-Champaign IL, 2007 Paris, 2013 Boulder CO), a workshop at *IHP Paris* and a one semester program in the *Fields Institute, Toronto*

PhD supervision and defenses

Two PhD theses written within the framework of the research projects supported by this grant were successfully defended with qualification *summa cum laude*: **Balázs Ráth** and **Bálint Vető** obtained their PhD degree in 2010, respectively, 2011. See their theses [Ráth (2010)], respectively [Vető (2011)] in section 2 above.

Three more PhD theses based (at least partially) on research results obtained within these projects are expected to be completed and defended in the foreseeable future (within one year): **Illés Horváth**, **Júlia Komjáthy** and **Anna Rudas** are close to finish their doctoral research.

Bálint Tóth (PI)