

We managed to reach the goals of the project. We achieved more than one hundred excellent results, 84 of them appeared already in the most prestigious journals of the subject, like *Combinatorica*, *Journal of Combinatorial Theory*, *Journal of Graph Theory*, *Random Graphs and Structures*, etc. We proved several long standing conjectures, solved quite old open problems in the area of hypergraphs and related subjects. Some of the problems were open for many years, sometimes for decades. It is not a direct research result but kind of an evaluation too that a member of the team became a member of the Norwegian Royal Academy and won Steele Prize. A summary of the results in the four years of the project is as follows.

2006

We worked out new methods to solve extremal hypergraph problems. By means of them, we proved sharp upper bounds on the total size of triangle-free hypergraphs. We also determined the maximum number of edges in hexagon-free very unbalanced bipartite graphs solving the extremal graph problem of Erdos, Sarkozy and Sos. The results are sharp and we found the extremal graphs too.

We determined the maximum number of edges in connected graphs not containing a path of given length and we found the interesting extremal graphs.

We determined asymptotically the necessary number of colors if we want to color the edges of a graph properly so that the sets of colors around the vertices should be different for any two adjacent vertices.

We solved the extremal graph problems for multipartite graphs with bounded degree if we find spanning transversals with given properties, like acyclic, no big component or no given subgraph.

We improved the multiplicative constant in the famous "crossing lemma" what implies improvements in other problems too.

We proved almost sharp upper bound on the number of edges of graphs that can be drawn in the plane with no self-crossing 4-cycle.

We proved properties of graphs when we know the distribution of the returns of random walks.

By means of topological methods, we investigated local and circular chromatic number of graphs.

We proved extremal theorems for ordered graphs not containing paths or cycles with given ordering.

We proved that the edge set of a bipartite graph can be decomposed into polylogarithmically many classes such that the ratio of the maximum and minimum degree is at most two.

We obtained interesting results on the Hall quotient of powers of graphs, which is a natural lower bound on chromatic number.

We proved sharp estimates on the number of edges in 3-uniform hypergraphs not containing a "book".

We asymptotically determined the Ramsey number of given odd cycle for 3-uniform hypergraphs.

We determined the maximum number of sums that can be made available so that we cannot determine the numbers. This result is important in storing data safely.

We found a new proof to describe the convex hull of two partite Sperner families.

We gave sufficient conditions for the splitting property of partially ordered sets.

We gave almost optimal degree conditions for the existence of 1-factors in k-uniform hypergraphs.

We proved the famous conjecture of Erdos and Folkman about complete sequences.

We generalized Dirac's theorem on Hamilton cycles for 3-uniform hypergraphs.

2007

We proved several results about special colorings of graphs with sets. Some of them are just the opposite to the conjectured statement and it increases the importance of the theorem.

We also proved the best possible result about adjacent vertex distinguishing, not necessarily proper edge colorings. The theorem we proved determines the precise number of colors we need to be able to find such a coloring.

We proved that the edge set of a bipartite graph of  $n$  vertices can be decomposed into  $\text{polylog } n$  almost uniform bipartite graphs.

We proved that every 28-cover of the plane by translated triangles can be partitioned into 2 covers. We proved several negative results about covers too, when there is no such a partition.

We improved the bounds on the number of edges of graphs having plane drawing without three pairwise intersecting edges.

We proved in the Propp model that the deterministic and random walks are very close to each other.

We studied the color distribution in complete bipartite subgraphs when we color special graphs like those of Kneser, Schrijver, Mycielski and others.

We developed a method to determine the biggest set system if we have conditions involving only containment relations. For example, when there is no set contained by  $r$  other sets, but there are other interesting special cases too.

We introduced a new graph parameter similar to the capacity of graphs. The importance of the parameter is shown by the fact that, by means of this parameter, we were able to prove new estimates in a problem of Korner and Malvenuto. We studied this new parameter and determined its minimum in graphs.

We studied the well-known theorem of Goldberg and West about cutting necklaces of pearls of different types into  $k$  pieces partitionable into two "equal" sets of pieces. We proved the best possible results when such a halving does not exist.

2008

We proved a conjecture of Faudree that a Hamiltonian graph with relatively small minimum degree contains a 2-factor of  $k$  cycles.

We finished and submitted the paper about colorings the vertices and the edges of a graph with color sets so that the color of an edge should be the symmetric difference of the color sets of its endvertices.

We proved asymptotically sharp bounds on the size of set families with no four sets such that the union of the first two is contained in the union of the other two.

We proved extremal type results on matrices motivated by database theory and coding theory.

We listed all 01-codes with pairwise difference exactly three.

We improved the theorem of Erdos, Frankl and Rodl on the number of  $L$ -free graphs where  $L$  is a fixed class of graphs.

We made combinatorial distinction between unit circles and straight lines considering how many coincidences they can have.

We found a new, short and transparent proof of the well known Foniok-Nesetril-Tardif description of finite duality pairs in the homomorphism poset of directed finite graphs.

We investigated the maximum size of the subsets of the vertices of a hypercube satisfying the property that the subspace (or cone) spanned by them will not intersect (contain) a cube.

Earlier, we proved a Dirac type theorem about Hamilton cycles in 3-uniform hypergraphs. Now, we extended it for  $k$ -uniform hypergraphs when the proof is much more involved.

We finished a paper about neighbour distinguishing colorings of the edges of a graph solving the problem completely.

We generalized the Hajnal-Szemerédi theorem for four-partite graphs.

We introduced and studied a new parameter of graph capacity type. We determined this parameter for several special classes of graphs.

We proved interesting results on the classical necklace bisection problem. We studied the case when one cut less is sufficient than in the worst case.

We studied sets of  $G$ -different permutations where  $G$  is a graph and  $G$ -different means that for any two permutations there exists a position such that the elements in this position of the two permutations are connected by an edge in  $G$ . We studied the maximal size of  $G$ -different sets of permutations.

2009

We studied the connections between the local chromatic number and the topological properties of graphs in several papers. We proved that there exist locally  $(t+1)$ -chromatic graphs among the topologically  $2t$ -chromatic ones but the local chromatic number of all strongly topologically  $2t$ -chromatic graphs is at least  $t+2$ .

We designed a new type of fingerprinting codes that asymptotically achieve the optimal rate, namely the fingerprinting capacity.

We proved tighter version of the renowned crossing number lemma for graphs that are far from regular.

We constructed a system of axis-aligned rectangles in the plane for any positive integers  $c$  and  $k$  satisfying the following property. For any coloring of the rectangles in the system with  $c$  colors there exists a point of the plane covered by exactly  $k$  of the rectangles, each receiving the same color.

We gave an algorithmic proof of the Lovasz Local Lemma. We give a simple polynomial time probabilistic algorithm to find the point in the probability space whose existence is claimed by the original lemma. It works in nearly all cases the original lemma is applicable.

We considered how the vertices of a graph can be colored if we insist that in the neighborhood of any vertex there must be a color used exactly once. We show that the needed number of colors is approximately the square root of the number of vertices if the vertex itself is not considered to be an element of its neighborhood, but a polylogarithmic number of colors always suffices if we consider neighborhoods containing the vertex itself too.

A  $q$ -ary equidistance 3-code is a set of sequences of  $n$  terms from the alphabet  $0,1,\dots,q-1$  such that every pair differs in exactly 3 positions. All such codes are determined.

We considered an infinite graphs whose vertex set is the set of natural numbers and adjacency depends solely on the difference between vertices. We study the largest cardinality of a set of permutations of  $1,2,\dots,n$  such that any pair of which differ somewhere in a pair of adjacent vertices of  $G$  and determine it completely in an interesting special case. We give estimates for other cases and compare the results in case of complementary graphs. We also explore the close relationship between our problem and the concept of Shannon capacity "within a given type".

We investigated the local chromatic number of shift graphs and prove that it is close to their chromatic number. This implies that the gap between the directed local chromatic number of an oriented graph and the local chromatic number of the underlying undirected graph can be arbitrarily large.

Korner and Malvenuto asked how many linear orderings (i. e., permutations) of the first  $n$  natural numbers such that any pair of them place two consecutive integers somewhere in the same position. This led to the notion of graph-different permutations. We extend this concept to directed graphs, focussing on orientations of the semi-infinite path whose edges connect consecutive natural numbers. Our main result shows that the maximum number of permutations satisfying all the pairwise conditions associated with all of the various orientations of this path is exponentially smaller, for any single orientation, than the maximum number of those permutations which satisfy the corresponding pairwise relationship. This is in sharp contrast with a result of Gargano, Korner, and Vaccaro concerning the analogous notion of Sperner capacity of families of finite graphs.

We applied a Borsuk-Ulam type theorem due to Tucker and Bacon to the classical necklace bisection problem. Generalizing a well-known theorem by Goldberg and West we show that even if the number of cuts available is one less than needed for fair division, then the two participants can prescribe which of them gets more of which types of beads. They can prescribe any positive number of types this way and still fairly divide the rest.

We continued the work begun by Anstee, Ferguson, Griggs, Kamoosi and Sali on small forbidden configurations. We define a matrix to be simple if it is a  $(0,1)$ -matrix with no repeated columns. Let  $F$  be a  $k \times l$   $(0,1)$ -matrix (the forbidden configuration). Assume  $A$  is an  $m \times n$  simple matrix which has no submatrix which is a row and column permutation of  $F$ . We define  $\text{forb}(m,F)$  as the largest  $n$ , which would depend on  $m$  and  $F$ , so that such an  $A$  exists. We determined this parameter in some interesting special cases.

We studied 3-color critical  $k$ -uniform hypergraphs. Lovasz proved an upper bound on the number of edges in 3-color critical  $k$ -uniform hypergraphs with  $n$  vertices. Now, we proved an ordered version that is a sharpening of Lovasz' result.

We investigated properties of coincidence ideals in subattribute lattices that occur in complex value datamodels, i.e. sets of subattributes, on which two complex values coincide. We let complex values be defined by constructors for records, sets, multisets, lists, disjoint union and optionality, i.e. the constructors cover the gist of all complex value data models. Such lattices carry the structure of a Brouwer algebra as long as the union-constructor is absent, and for this case sufficient and necessary conditions for coincidence ideals are already known. In this paper, we extend the characterisation of coincidence ideals to the most general case. The presence of the disjoint union constructor complicates all results and proofs significantly. The reason for this is that the union-constructor causes non-trivial restructuring rules to hold. The characterisation of coincidence ideal is of decisive importance for the axiomatisation of (weak) functional dependencies.

We proved nice results on the 3-colored Ramsey number of odd cycles

We are preparing a list of probably three papers solving the famous Erdos-Sos conjecture on the maximum number of edges in a graph of  $n$  vertices not containing a given tree of  $k$  vertices.

We studied the algorithmic aspect of a recently considered reconstruction problem, and developed a linear time, bounded sized query algorithm. This differs greatly from the Dress - Erdos algorithm for the Schutzenberger - Simon problem, since this is not an Oracle type algorithm.)

We investigated the so-called  $M$ -part Sperner families. The archetype of the problem is Paul Erdos' famous  $k$ -Sperner systems. We achieved several results (including, among others, the determination of the convex hull of this new set system class and the description of all optimal systems for a wide region of the parameters). We also surveyed all available connected results, and introduced two brand new, closely related problems: the problem of  $k$ -fold  $M$ -part Sperner property and the problem of the existence problem of full transversals.

We proved extremal theorems on paths of given length in  $k$ -uniform hypergraphs. The results are very different for different definitions of paths.

We proved a conjecture of Faudree et al. about the degree condition for the existence of 2-factors of  $k$  cycles in Hamiltonian graphs.