

Non-classical interrelations in social phenomena

Final Report

December 27, 2023

1 Theoretical results

One of the most important theoretical results is the paper Koniorczyk et al (2020). In this paper we have studied the relation of Bayesian game theory to the theory of quantum nonlocality. We have analyzed the difference between ex ante and ex post equilibria in classical games played with the assistance of a nonlocal (quantum or no-signaling) resource. We gave a constructive procedure for designing Bayesian games where the players can benefit from the use of a resource providing no-signaling nonclassical correlations.

Another important theoretical results is Pintér (2022). Taking advantage of ambiguity in strategic situations is well documented in the literature. However, so far there are only few results on how to make ambiguous strategies. In this paper we introduced a procedure which makes objective ambiguity, concretely it draws an element from a set of priors, defined by a belief function, in a way that it does not lead to any probability distribution over the priors. Moreover, we define the notion of ambiguous strategy, and by means of examples we show how to make ambiguous strategies in games. It is not trivial, but ambiguity is related to non-classical physics, hence the understanding ambiguity, which is well-documented in social phenomena, can help to find a common umbrella for non-classical physics and incomplete information games.

Further related papers are Hellman and Pintér (2022), Dornai and Pintér (2022) and Bartl and Pintér (2023). The first paper is a contribution to the so-called type space literature (which seems to relate non-classical things) and the last two are contributions to the TU-games literature. The TU-games are transferable utility cooperative games, non-additive probabilities are special TU-games, and non-additive probabilities are applied to model ambiguity (its relation to our research has been discussed earlier).

The far most important theoretical result comes from Koniorczyk et al (2020). It is a corollary of the fact that there is the connection between the non-locality known in non-classical physics and incomplete information games (Brunner and Linden, 2013). Our conclusion is the the connection is, actually, not between the so-called Bell inequalities (Bell, 1987) and Bayesian games (Harsányi, 1967-68), but rather between the underlying sheaf theoretic framework of non-classical physics (Abramsky and Brandenburger, 2011) and the so-called type spaces (see e.g. Samet (1998)). A new research on this topic will start from this point on January 1, 2024.

2 Applied results

Our applied results are twofold. First, we have been working on quantum optimization, that is, using non-classical resources, e.g. quantum computers, in optimization. This is a very hot topic now. Second, we have developed an emulator to generate non-classical correlations. We devote a separate section for the second result.

We have investigated practical applications of quantum annealing. Notably we have demonstrated the potential usefulness of quantum annealing technology in railway optimization problems for the first time (Domino et al, 2023). Our first proof-of-concept demonstration has been extended to a bigger-scale model tested in real-life situations, which may bear practical relevance (Koniarczyk et al, 2023). This line of research was extended to another industrial problem (Śmiechrzalskia et al, 2023), and also lead to a contribution to methods for evaluating quantum annealers' results in general (Domino et al, 2022).

We have solved problems in coding theory (Hamming packings) with the goal of providing additional practically relevant benchmark problems for quantum annealers in the near future (Naszvadi and Koniarczyk, 2023). As for quantum devices, we have achieved more theoretical results which are relevant for quantum random number generators (Ortega et al, 2023).

3 Non-signalling emulator

We have designed and implemented a web service that implements nonlocal no-signaling correlations in a logical sense (Koniarczyk et al, 2022). We have discovered for the first time the usefulness of using no-signaling boxes in different causal orders by the parties by introducing a protocol that can convert extremal two-party-two-input nonlocal no-signaling boxes of any type into any other extremal two-party-two-input nonlocal no-signaling box perfectly, which requires such a use of non-signaling boxes (Bodor et al, 2022).

No-signaling correlations were discovered along the study of quantum nonlocality. The Einstein-Podolsky-Rosen paradox, originally a strong philosophical criticism of quantum mechanics was quantified by John Stuart Bell in the 1960-s in the form of his celebrated inequalities. The early experiments by Alain Aspect and John F. Clauser were followed by ones improved thanks to the huge development of lasers and experimental quantum optics at the end of the 20th century, conducted by Anton Zeilinger. These three experimentalists were awarded 2022's Nobel-prize in physics for this activity. Today this 'spooky action at a distance' is considered as a key ingredient of quantum cryptography and communications. No-signaling correlations form a superset of quantum correlations.

In general no-signaling correlations can coordinate parties without facilitating communication. Some of them can be realized with quantum technology but still not in a straightforward or broadly accessible manner. It is certainly a valid and relevant question to ask what would happen if they were easily accessible, in what kind of situations people would benefit from their use. And so is the question of whether such correlations arise in other, non-physical phenomena where multipartite conditional probabilities appear. A step towards the study of these research questions is the introduction of the notion of

the logical implementation of no-signaling correlations. The physical devices that realize quantum correlations require sharing entangled quantum systems in advance, and carry out selectable measurements on these. This results in an implementation in which the parties do actually not interact when the correlations are realized; a relevant feature in cryptography applications. If, however, we allow the devices showing nonclassically correlated behavior to communicate a central server, the users of these devices can experience the same behavior, and their devices alone would still not be applicable for communication. From the physicist's point of view, in the case of quantum correlations, this is an emulation of the quantum experiments. From the general user's perspective, however, it can be regarded as a logical implementation. And such an implementation is possible for any no-signaling correlation, including supra-quantum ones.

In our project we have designed and implemented a state-of-the-art implementation of nonclassical correlations via a RestFUL Web API service. We have introduced the algorithm which serves as the basis of the implementation. As a side result, we have developed skills and software to detect nonclassical correlations in any bipartite conditional probability distribution. We have designed and implemented a computer system to provide no-signaling correlations as a web service. As the chosen technology is very prevalent, the introduction of our API, apart from facilitating the experimentation with no-signaling correlations for those who have very basic programming skills, serves as a technological basis for implementing user applications that rely on nonclassical correlations.

Beside the obvious technological and educational aspect, such an opportunity paves the way to the experimental study of the application of nonclassical phenomena in social science experiments: devices can be implemented and given to the subjects of the experiments. Currently, as a continuation of our work, a smartphone app is being developed by us on this basis. Our service is openly available for research purposes after a registration at <https://wigner.hu/nonlocalbox>. For the raw random number generation, the service uses a commercially available quantum random generator, so it includes real quantum technology as a demonstration.

We have conducted useful discussions with experimental experts outlining the possibilities introduced by our API. Most manifestations of nonlocal nonclassical correlations can be associated with Bayesian games (as we describe this systematically in one of the publications resulting from the present project). Consequently the members of the physics community often describes these phenomena in terms of situations with parties who learn the state of random factors and then make decisions. In spite of all these, from the discussions we have found that in the context of social sciences, to design such experiments with actual players is a significantly bigger conceptional challenge than we expected. The bottleneck was thus not the availability of the device outside physical experiments, but the lack of understanding the real effects of such correlations in human situations. The situations described in the literature are too theoretical and premature for experiment designs. Our development makes the correlated devices readily available, hence, the first step has been made. The actual experimentation was found to be beyond the scope of the present project, especially as this branch of our research was also delayed by the COVID pandemics.

4 Bodor's PhD

One of us, András Bodor received his PhD as member of this project: Bodor (2022).

The details: András Bodor: On the Nonclassical Behaviour of Unipartite and Bipartite Systems (2022)

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Supervisor: Mátyás Koniorczyk

Abstract: The results presented in this thesis can be divided into two groups. Concerning unipartite quantum systems we have presented an alternative view of quantum state discrimination, while for bipartite systems we have considered the connection between nonlocal correlations and game theory.

In the field of quantum state discrimination we have introduced a new approach by using ROC (Receiver Operating Characteristic) curve which is used in classical statistics for characterizing an ensemble of discriminators. We have introduced its quantum variant and examined its properties. As a result we could relate the curve to several quantum properties and quantitative features, gaining an expressive and intuitive picture about their correspondence. Additionally, we have found a quantity whose definition naturally follows from the shape of the ROC curve: the quantum-Bhattacharyya coefficient. This can serve as a similarity measure for quantum states.

In the second part of the dissertation we have investigated the correspondence between two-party nonlocal correlations and Bayesian games. We have introduced a systematic method for generating games with nonlocal advantage that covers most of the cases found in the literature. We have analyzed in detail the structure of the local and no-signaling polytopes in case of two- input-three-output Bell situations, including the illustration of our method. Using different notions of game theoretical equilibrium we have uncovered certain structural properties of Bell inequalities.

Meanwhile, an exception is also found: the correlation discovered originally by Vértesi and Bene. We have found that their inequality is a basis for a game which has unusual equilibrium properties.

5 Events

Our group were also engaged in organizing scientific events mainly, workshops. These workshops were partially or fully done in the framework of this projects. The list of the events is as follows:

- Quantum Decision Theory Workshop, January 3-4, 2018, Pécs
<https://upgtrg.ttk.pte.hu/home/events/>
- Advances in Stochastic Games, January 23-24, 2019, BME, Budapest
<https://www.bme.hu/node/6454?language=en>
- Meeting on Games, Stochastics, and Dynamics, October 6-7, 2021, BME, Budapest
<https://sites.google.com/view/stody-games2021/főoldal>

- Kvantuminformáció és optimalizálás, 2021 október 26-27., Pécs
<https://tab.mta.hu/pecsi-teruleti-bizottsag/esemenyek/kvantuminformacio-es-optimalizalas>
- Three faces of game theory, September 15, 2022, Corvinus University of Budapest
<https://www.ccor.hu/?p=23>
- Dynamics and Decisions (D&D), October 27, 2022, Corvinus University of Budapest
<https://www.ccor.hu/?p=55>
- CCOR Workshop on Three Faces of Decision, May 24 2023, Corvinus University of Budapest
<https://www.ccor.hu/?p=172>
- CCOR Workshop on Information, Games and Decisions (IGD), May 31 2023, Corvinus University of Budapest
<https://www.ccor.hu/?p=182>

6 Short summary of the results

Publications: We have published 9 papers in international journals, and 2 in a Hungarian journals. Furthermore, we have 5 submitted papers (already uploaded to arXiv). In addition to a PhD thesis.

Conference talks: We haven't listed the conference talks by the project members. Those are at least 3 dozens, approximately 2 dozens are in international conferences.

Events: We have organized, or we were involved in the organization of 8 workshops.

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