Scientific summary

(PD127982)

Description of strongly coupled quantum systems with functional methods

1. Introduction:

The main goal of the project was to obtain new results on various strongly interacting quantum systems, where conventional mathematical methods, such as perturbation theory, break down. The proposed technique, being in the center of the research plan, was the Functional Renormalization Group (FRG). This method generalizes the concept of the Wilsonian Renormalization Group, and provides a functional differential equation that describes the scale evolution of the full quantum effective action. Throughout my research I applied this flow equation to various problems in particle physics and statistical mechanics.

Scientific results of the project can be divided into 4 parts: 1.) Gauge theories and the functional renormalization group, 2.) Topological phase transitions in the modified XY model, 3.) Effective 't Hooft and Yukawa couplings in meson models, 4.) Chiral phase transition in the three flavor strong interaction.

The results were materialized in 8 papers, all of which are published in peer reviewed journals (at the time of writing this summary the last 2 have already been accepted for publication but not published yet). The number of papers have reached the amount that was promised at the time of the application, the research plan has been successfully accomplished.

In what follows, I report on all new scientific achievements in detail.

2. Scientific achievements

2.1 Gauge theories and the functional renormalization group

The main goal of this part of the project was to develop an approximate scheme of the FRG flow equation, which can respect gauge symmetry. This is highly nontrivial in the sense that it is well known even from perturbative calculations that the introduction of any momentum cutoff into loop integrals breaks gauge symmetry.

First, I performed an investigation in the U(1) gauge theory with global O(N)+O(N) symmetry [1], as all violations of gauge symmetry turn out to be simpler if gauge symmetry is Abelian. This served as a first step toward the non-Abelian cases. I have showed that for any number of N, gauge symmetry violation due to the momentum cutoff can be cured via appropriate choice of the gauge fixing parameter of the R_ksi gauge fixing. The results revealed that new IR stable fixed points with nonzero gauge coupling can exist in the system, which, as a nice byproduct, hinted that in dislocation dynamics, a quantum solid-to-hexatic melting phase transition could be of second order in two spatial dimensions.

Second, the previous investigations were extended to general multicomponent scalar theories, again with U(1) gauge symmetry [2]. I examined the necessary and sufficient conditions for the existence of certain classes of theories, which are consistent in the so-called local potential approximation (LPA) of the RG flows, in the sense that the flows close with respect to a set of operators defined at the ultraviolet (UV) scale. As one of the main results, I showed that in the aforementioned (consistent) classes of theories, the R_ksi and standard covariant gauge fixings are equivalent. Surprisingly, this statement is highly non-trivial, as it turned out that in order to show equivalence of the two gauge choices, the infrared regulator functions need to be altered and mapped into each other carefully. In other words, if one switches gauges, one also need to switch IR regulators. In the same context, the regulator modified Ward-Takahashi identities were also analyzed and shown to be consistent with the RG flow equation.

Third, building upon the previous results, eventually I turned my attention to the case, where the gauge group is non-Abelian, in particular SU(N) (with an emphasis on N=3) [3]. The FRG gauge symmetry violation was reanalyzed and I showed that the previously discovered adjustment of the gauge fixing parameter (in ordinary covariant gauge), with only a minor modification, can also be naturally implemented in the non-Abelian scenario. Therefore, loss of gauge symmetry at infrared scales can be circumvented even in models with gauge symmetries other than U(1). As an application, I calculated in three spatial dimensions the beta function of the gauge coupling of scalar QCD [and in general for the SU(N) gauge group] with U(n)xU(n) global symmetry (n being the flavor number), which for n=3 is the Ginzburg-Landau theory for color superconductivity. The main result of the calculation was that there is not any IR stable fixed point in the system, therefore, as opposed to ordinary superconductivity, the color superconducting phase transition cannot be of second order, and is always (presumably) of first order.

2.2 Topological phase transitions in the modified XY model

The XY model is a two dimensional spin model, where at each lattice site an angle variable is defined. The model is known to be the most prominent example of a system, where at finite temperature a topological phase transition emerges. In such transitions, topological defects (which are non-perturbative spin configurations), called vortices, are in the center of attention. It turns out that at the low temperature phase of the system such objects are confined into pairs (vortex-antivortex), which can become free as the temperature increases. The transition cannot be accessed via perturbation theory, and the original description was done via a rather unconventional real space renormalization group.

First, I managed to reproduce the description of the transition via the FRG, showing that the method does give account of topological vortex configurations, which clearly demonstrates its non-perturbative nature. As a next step, I did investigations in the modified XY model [4], where in addition to the usual next-nearest-neighbor spin-spin interaction, a Pi periodic term with tunable interaction strength is also added into the Hamiltonian. By tuning the interaction strengths, using the FRG flow equation, I mapped out the phase structure of the system and showed that there can exist a two step phase transition, 1) related to half vortex-antivortex unbinding, and 2) the emergence of vortex-antivortex molecules. This was also confirmed by Monte-Carlo simulations [4]. Even though this study was not planned in the original research proposal, it has clearly advanced possible implementations of the FRG method, which was completely in line with the methodological aspects of the research plan. Furthermore, these findings may have opened up new possible research directions, presumably related to describing topological phase transitions from a renormalization group point of view.

2.3 Effective 't Hooft and Yukawa couplings in meson models

To this day, the fate of the U(1) axial anomaly of quantum chromodynamics (QCD) at finite temperature and/or density is controversial and under debate. Many lattice simulations predict that the U(1) axial symmetry is still broken beyond the (pseudo)critical temperature (T_c) of the chiral restoration, but around equally many studies claim that it recovers as chiral symmetry restores. In three flavor effective models of QCD the anomaly is taken into account via `t Hooft's determinant term containing 3-point vertices of meson operators. In principle, coefficients of all operators in the Lagrangians, including the aforementioned determinant term, are considered to be (coupling) constants, without any field or environment dependence. In the quantum version of the action, however, fluctuations introduce temperature (T), baryochemical potential (μ_B), and also field dependence, as the couplings become coefficient functions. In [6] I calculated using the FRG formalism the field dependence of the coefficient function of the anomaly at the lowest order in a field expansion of the O(18) chiral invariant. This basically showed how the strength of the anomaly changed as the chiral condensate varies (e.g. due to finite temperature and/or density effects).

Somewhat counterintuitively, I found that as the chiral condensate gradually evaporates with the temperature (T), the U(1) axial anomaly increases. If true, this can only work for T < T_c, but definitely not when T >> T_c, since it is well-established theoretically that the axial symmetry has to recover at high enough temperature due to Debye screening of the instanton field. Since these arguments become valid way beyond the critical temperature, nothing prevents an effective theory, which is thought to be valid below the critical temperature, to show the opposite behavior. As an application, I also calculated how the anomaly changes at the nuclear liquid-gas transition. The main assumption was that once nucleons are coupled to the meson model, they gain their mass solely from the chiral condensate via a Yukawa coupling. As suggested by the behavior of the anomaly function I found that as chiral symmetry partially recovers at the transition point, the anomaly strengthens.

These results were then extended in two directions [7]. The first thing was to produce a fully non-perturbative treatment in regard to the 't Hooft anomaly coupling, without performing any expansion in loop integrals. The second was to introduce finite temperature instanton effects. Since in an effective model gauge fields are not present, the way one can incorporate topological fluctuations into the model is to give explicit temperature dependence to the bare anomaly parameter, which is defined at the UV scale of the theory. This dependence was borrowed from the dilute instanton gas approximation, which predicts an exponential dumping of the anomaly at high temperature. Results show that the earlier, naive calculations are qualitatively correct, the anomaly strengthens with respect to the temperature, but due to the instantons eventually it does vanish as the temperature goes beyond the critical point. The interplay between the mesonic and topological fluctuations shows that the axial anomaly is unlikely to be practically restored even at approximately 200 MeV, which is around ~1.3 T_c.

Motivated by the previously mentioned analysis of the nuclear-liquid gas transition, I also performed an investigation of the non-perturbative renormalization group flow of the Yukawa coupling in the same meson model, now extended with fermions [5]. As already mentioned, the Yukawa term plays a central role in generating fermion mass, and similarly to the anomaly coefficient, the Yukawa coupling in principle also develops condensate dependence, which was extensively analyzed in the literature through methods, which, however, were only developed for two flavor chiral symmetry.

The main motivation and ultimately the result of the study [5] was that the methods that were valid for two flavors become unusable for three flavors. The reason is that three flavor chiral symmetry allows for a much richer set of chirally invariant operators as opposed to its two flavor counterpart, therefore, earlier methods basically mix up contributions that should have been projected out once one is interested only in the field dependence of the Yukawa interaction strength. In [5] one I proposed a new strategy to distinguish between the field dependent Yukawa coupling and those interactions that are non-Yukawa like and are related to higher order chiral invariants of the U(3)xU(3) group of chiral transformations. The new method allows for more consistent re-analyses of the finite temperature and density behavior of the system.

2.4 Chiral phase transition in the three flavor strong interaction

The order of the chiral transition in QCD with vanishing quark masses remains under debate. By analyzing a Ginzburg-Landau type potential, it was predicted long ago that for three flavors, the transition has to be of first order, induced by mesonic fluctuations. For two flavors, however, the transition was expected to be of second order [belonging in the O(4) universality class], if the axial anomaly does not vanish and is large enough at the critical point. If the axial U(1) symmetry does get restored, then the transition is of first order. These analyses were based on the epsilon expansion of the renormalization group flows, and in the past 10 years, there has been increasing evidence that for two flavors, the transition has to be of second order after all, irrespectively of the anomaly evolution. For three flavors, however, the original argument seemed to survive, it was expected that the transition is of first order. Last year, new lattice results appeared in the literature claiming that no first order transition is seen approaching the chiral limit. If these results are true, then there is a conflict between lattice results and the RG analysis.

My main motivation was to resolve the conflict between RG and lattice results, which I managed to achieve [8]. For the first time in the literature, I calculated all the RG flows of the relevant couplings directly in three dimensions, without using the epsilon expansion. This turned out to be rather tedious, as in three dimensions, there are 9(!) relevant or marginal operators. Using the FRG method, I calculated all the 9 beta functions and searched for infrared stable fixed points in the system. The main result was that as opposed to close to four dimensions, and extrapolating the results to three dimensions via the epsilon expansion, there can exist a new fixed point with one relevant direction in the infrared. This, however, only describes a thermal transition, if the axial anomaly vanishes at the critical point. Still, the results showed that i.) the chiral transition can be of second order for three flavors, ii.) there is not necessarily a conflict between lattice results and the RG analysis, and iii.) the epsilon expansion is invalid when applied to the chiral transition.

3. Summary

In summary, the most important pieces of the research achievements of the project can be separated into the following points.

- It has been shown that U(1) gauge symmetry and the local potential approximation of the FRG flows can be compatible with each other by choosing appropriate gauge fixing, for various multicomponent scalar models [1-2].

- The previous results have been extended to the SU(3) case, where it has also been shown that the color superconducting phase transition cannot be of second order [3].

- In the 2d XY model it has been shown that the FRG method can efficiently describe topological phase transitions, and in modified version of the model it revealed a new, two-step transition at finite temperature [4].

- In the three flavor meson model it has been found that the 't Hooft coupling, describing the axial U(1) anomaly, can show an intermediate strengthening behavior with respect to the temperature before reaching the critical point [6-7].

- The previous analysis has been extended to finite chemical potential and it has been shown that if the nucleon mass is generated through Yukawa interactions, then at the nuclear liquid-gas transition point the anomaly strengthens [6].

- The field (condensate) dependence of the Yukawa interactions has also been reanalyzed in the context of the three flavor meson model and it has been shown that earlier analyses cannot be generalized to three flavors, thus a new computational method is now proposed [5].

- It has been demonstrated via an explicit three dimensional RG computation that the finite temperature chiral phase transition of QCD can be of second order for zero quark masses, accommodating RG arguments with recent lattice results [8].

These results are published in the following papers.

[1] A. J. Beekman and G. Fejos, Phys. Rev. **D**100, 016005 (2019).
[2] G. Fejos and T. Hatsuda, Phys. Rev. **D**100, 036007 (2019).
[3] G. Fejos and N. Yamamoto, JHEP 12, 069 (2019).
[4] M. Kobayashi, G. Fejos, C. Chatterjee, and M. Nitta, Phys. Rev. Res. 2, 013081 (2020).
[5] G. Fejos and A. Patkos, Phys. Rev. **D**103, 056015 (2021).
[6] G. Fejos, Symmetry 13(3), 488 (2021).
[7] G. Fejos and A. Patkos, arXiv:2112.14903, accepted for publication in PRD
[8] G. Fejos, arXiv:2201.07909, accepted for publication in PRD