Final report

on the K-17 NKFI No.: 124859 research program

Mechanics of masonry-, no-tension structures

(summary)

1. Research work and the results

The research program has two main parts: i) application and examination of the thrust line, thrust surface method for masonry arches and domes, ii) homogenization method applied for the determination of failure and deformational characteristics of masonry elements. There are experimental works related to both parts.

1.1 Research on the thrust line method

1.1.1 Masonry arches

Based on the definition of the thrust line, we proposed earlier to consider the possible stereotomies of the arch, different shapes of masonry arches were examined considering the Heymanian assumptions which results only rotational failure mechanisms, otherwise considering the possibility of slip too.

The developed model considers continuously variable stereotomy function with geometrical constraints in order to be realistic in the sense of buildability of the arch geometry. The finite friction, i.e. the possibility of slip mechanism, is also considered in the model as a geometrical constraint. This way the minimum thickness calculation of the arch become a constrained optimization problem.

First circular masonry arches were studied. It was found that the minimal thickness of the arch has an upper limit. It is always possible to construct a suitable stereotomy for thinner arches. Even zero thickness may be found for extreme (i.e. non-realistic) stereotomy. Some of our results were confirmed by other researchers using different model: *N.A. Nodargi- P. Bisegna: Thrust line analysis revisited and applied to optimization of masonry arches. Int. J. of Mechanical Sciences 179 (2020) 105690.*

Circular pointed masonry arches were also studied. It was shown that by the help of the envelope of the all possible thrust lines (i.e. catenary type thrust line) it is possible to decide in advance that the rotational failure mechanism is due to 5, 6 or 7 hinges, i.e. the geometry of the circular pointed arch defines the failure mode. However, the catenary type thrust line gives the lower bound of the stereotomy dependent minimum thicknesses only in case of 5 hinge mechanism. It was also shown that circular pointed arches may be failed only by maximum number of 7 hinges, and it means an optimum in the sense of minimum thickness. Examination of French and English gothic transversal arches did not prove any tendency toward the optimal (i.e. thinnest or 7 hinge) geometry.

The results support Poleni's stereotomy independent, orange slice type examination of the St. Peter Chatedral in Rome.

It was also examined how the frictional resistance (i.e. possibility of slip) between voussoir elements influences the minimum thickness of the masonry arches. The possibility of slip mechanism substantially increases the minimum thickness value. In case of circular masonry arches the radial stereotomy, as the most important one from the practical points of view, gives safe minimum thickness for any possible stereotomy. However, for elliptical masonry arches this is true only if the axis ratio of the ellipsis is less than 1,8. For larger axis ratio possibility of slip mechanism must be considered for a safe minimum thickness.

It was also a question if there are such arch geometries where the number of hinges, developing at failure, is more than 7, being arbitrary odd number. It was found that there such convex arch geometries which fail by more than 7 hinges. For those arches higher number of hinges results smaller minimum thicknesses, but higher number of hinges is not a necessary condition for smaller minimum thickness.

It is important to point out that the bounding values of the minimum thickness are always exist for any arch geometry, but the catenary type thrust line does not necessarily provide any bound.

Experiments on arches

Dry joint arches were produced to make experiments on arches. Wood was the material of the voussoir elements. The arch geometry was circular. In two cases they were half circle (span: 1,90 m, radius: 95 cm, thickness: 19 cm), and there were two different pointed circular arches (spans: 85cm, 152 cm, radius: 95 cm). The stereotomy was radial, except for one circular arch where vertical stereotomy was used in the middle of the arch, and it was gradually changed towards radial position near springing. A testing rig was built which made possible to move the supporting point towards or apart from each other (max. 29 cm) while the horizontal thrust force was measured, together with the support movement and the vertical displacement of the top mid-point of the arch. Some markers were put around the possible location of the opening gap (i.e. hinge) in order to measure the voussoir elements movement digitally (Fig. 1). For each geometry a formwork was used to build the arch. The friction coefficient between the elements were large, which prevented the possibility of sliding failure. In order to provoke sliding failure too polyethylene foil was used to reduce the friction coefficient.

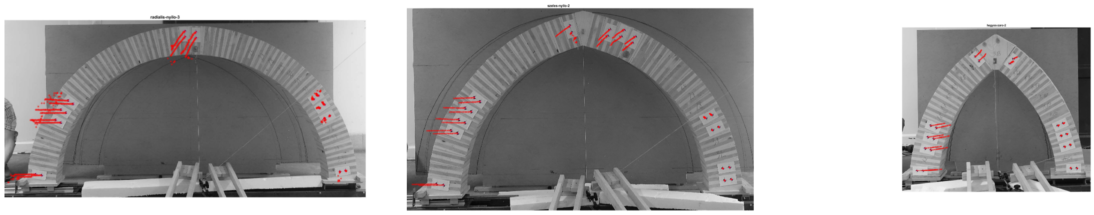
The three different arches with radial stereotomy were easy to build on the formwork, and they were standing stable because the thickness to radius ratio ($t/R = 0{,}2$) was larger than the theoretical minimum thickness ($\min(t/R) = 0{,}1074$).

In each cases the failure mode was symmetric except when sliding and rotational failure modes were combined, that resulted asymmetry in failure mode. The support movement capacity of the arches was large. It was measurable to the arch thickness or even larger.

In case of the rotational failure mode the mid hinge locations were changed during the loading process, closing the opened gap and opening the next one. During this process there was such a state of the arch when both gaps were opened and the top and bottom arch parts are connected to the voussoir element along edges only. In this state the element had some small vibration. After increasing the support movement, the element jumped in a stable position closing one gap. Finally, the collapse was sudden at very large displacement.

In case of development of sliding the collapse was preceded by a continuous, slow motion without support movement. This is due to the difference between static and kinetic friction coefficients.

Negative experience was that the arch with partly vertical stereotomy could not stand without formwork, although theoretically it is possible. It is not enough to provide large friction coefficient between the elements, but considerable forces are also needed on the contact surfaces. These forces are not developed in a standing alone arch due to self-weight in case of vertical stereotomy. That force must be introduced externally, e.g. applying filling above the arch. This kind of force introduction technic is applied for flat brick lintel by the mason too.



a) circular arch with opening the span *b) large pointed arch with opening the span* *c) small pointed arch with closing the span*

***Figure 1**: Arch experiments. Left support moves. Red lines are showing the displacement trajectory of the marker during the support movement*

1.1.2 Masonry domes

It is characteristics for spherical masonry domes that they may crack meridional due to loading, even for self-weight. We had two main questions to answer: i) Is there any possibility to avoid meridional crack of the dome? ii) Is it possible to prove that a meridional cracked dome is safe in cracked state?

Based on membrane shell theory it was found that there is such thickness variation which results no tension in circumferential direction of a dome. However, the thickness variation is singular at the top and at the springing of the dome. Top singularity can be eliminated by building a hole at the top (i.e. cutting out singular part) or simply using a non-singular thickness function at the top part. The singularity at the springing cannot be eliminated because without that there is always circumferential tension which may cause cracking. The intention of the master builders of past ages, that the dome thickness was increased towards springing was right but unfortunately the goal cannot be reached for the thickness variation near springing is unrealistic.

Meridionally cracked dome was examined by bending shell theory. The top part, compressed in both meridional and circumferential direction, was considered as a membrane shell. The cracked bottom part is considered as a bending shell which has no resistance in circumferential direction, i.e. cracked meridionally: The so called smeared crack model was used in the framework of the bending shell theory. This model is statically determinate in the sense that only the equilibrium equations are enough to find solution.

It was found that the thrust surface of the cracked part is a general one, because the balancing forces are not acting on the tangent plane of the thrust surface although the thrust surface is moment free. Rotational failure mechanism of the dome is possible only when the crack runs up into the compressed cap when considering different length meridional cracks. This is possible if the crack propagation is unstable. By the help of the model it was proved that the 300 years old, cracked masonry dome of Gol Gumbaz, India is safe in cracked state considering the reported crack length.

The model developed for finite number, but rotationally symmetric discrete crack system gave the same result as the smeared crack model if the number of cracks goes to infinity. Otherwise the examination of the problems of the model gave the basis for the conclusion: development of the meridional crack system of a dome happens in a rotationally non symmetric way. This was also supported by XFEM based numerical simulation (although there were a lot of technical problems while performing the calculation).

An approximate, radially spring supported, and radially loaded ring model was developed, which assumedly gives information about the crack development of the dome. Springs substitute the stiffness of the top dome part, and the ring geometry model the dome geometry. Elastic but brittle behavior was assumed for the ring material. The model showed rotationally non-symmetric crack development although the initial structural configuration was perfectly rotationally symmetric. The numerical results showed that after one crack was developed other six appeared on an increased load level, in a non-rotationally symmetric way. The resulted ring pieces have not the same arch length, although the difference was not too large. i.e. the developed crack system is not rotationally symmetric. Subsequently increasing the load new cracks are appearing reducing the arch length differences but not eliminating. The numerical results imply that for a material having finite resistance the number of cracks are also finite showing rotationally non-symmetric geometry. This corresponds to the real life experience.

1.2 Research on masonry elements

The masonry element is made of several masonry unit length layers with mortar joints, according to the actual bonding pattern. This is loaded experimentally to determine the strength and deformational parameters of the element in an experimentally homogenized way. These data are used to design masonry walls in the practice. Obviously experimental possibilities are limited and sometimes technically hard to perform, e.g. performing shearing test on masonry element.

The homogenization method gives possibility to perform simple but effective simulation for masonry elements to determine its deformational properties and strength. Even if the masonry unit and mortar are isotropic the masonry element behavior is orthotropic.

An engineering type homogenization procedure was developed, based on the rule of mixtures, for both head joint filled and not filled with mortar cases, to determine the orthotropic deformational properties. While for mortal filled head joints there are, not too sharp, upper and lower bounds for the different deformational properties, for the unfilled head joint case there is only an upper bound. (The unfilled head joint technology used more frequently nowadays.)

In the homogenization procedure the micro level is the level of masonry units and mortar, the macro level is the level of masonry element. The macro parameters are the average values of the micro parameters for both stress and deformation. The model uses the representative volume or unit volume for the calculation considering the periodicity of the bonding pattern. Using the equilibrium equations, material laws for each parts and a simplified deformation state on the micro level results the macro deformational properties. The failure of the parts at micro level defines the failure at macro level too. The method works with acceptable error compared to finite element results of masonry elements.

(The material law and failure criterion of the masonry unit are based on experiments. Masonry unit can be either isotropic with Huber-Mises-Hencky and Tresca failure criteria for compression and tension or anisotropic with Tsai-Wu type failure criterion. For mortar isotropic behavior with Huber-Mises-Hencky and Tresca failure criteria was assumed.)

We could proof numerically the experimental experience that compressive strength of the masonry element with unfilled head joints is the same as the strength of masonry element with filled head joints.

The proposed model, combined with simple experiments on masonry units gives possibility to determine macro characteristics of masonry elements by simple numerical simulation.

Experiments on masonry units

Laboratory experiments are needed to determine the deformational and strength characteristics of masonry units in order to perform numerical simulations for masonry elements.

Compressive strength and deformational modulus are needed in the direction of each sizes of the masonry units, i.e. perpendicular and parallel to bed joints (i.e. parallel to wall plane) and perpendicular to the wall plane too. Only the compressive strength perpendicular to bed joint is known and used in the everyday engineering practice, although the others are as important (e.g. in case of earthquake).

Solid masonry brick and hollow, fired clay blocks were examined. Each were prepared by mortar caps according to the testing standard. Six pieces of each type of masonry units were used for each experimental setup, altogether 36 for the two different masonry units. The compressive force, the vertical movement of the loading platen, and deformation on the side plane of the loaded element by digital image correlation technic were measured. The latter one gave more reliable data for local deformation than the strain gauges used for the trial experiment.

In case of the solid brick, for perpendicular to bed joint position, the confining effect of loading platen is very strong, because of the small height in loading direction. The failure is according to an envelope shape as it is experienced on concrete cubes too. For the other loading positions, the failure is due to cracks parallel to compression and buckling of the parts between cracks. The strength of the unit is varied according to the loading direction: compressive strength perpendicular to bed joint : parallel to bed joint : perpendicular to wall plane = 1 : 1,14 : 0,55. The data suggests that stretcher bond application of the solid brick is more advantageous.

In case of the fired clay block, for perpendicular to bed joint position, the confining effect of loading platen is not too large, because of the larger height in loading direction. For each loading positions, the failure is due to cracks parallel to compression and buckling of the parts between cracks or due to the collapse of the internal rib system of the block. The strength of the unit is varied according to the loading direction: compressive strength perpendicular to bed joint : parallel to bed joint : perpendicular to wall plane = 1 : 0,025 : 0,5. The data suggests that header bond application of the block is more advantageous.

It is a general experience that the displacements of the element have large variation and so to estimate deformational modulus statistical analysis is needed.

The Athena non-linear finite element software was used to simulate the experiments and determine the mechanical parameters of the fired clay material. The material model of the software is originally for concrete, but because the fired clay like concrete is a quasi brittle material it can be used to simulate the behavior of the masonry units. The global behavior of the masonry blocks in each direction can be reproduced quite fairly. In case of solid bricks, the confining effect of loading platen gives some problem to solve.

The numerically determined fired clay parameters are used for simulate loading cases, hard to perform experimentally, to determine a Tsai-Wu type failure criterion of the masonry units. This is needed to model the masonry element.

2. Difference in planned and completed research plan

During the first two years the research was according to the work plan. In the third year the pandemic slowed down the progress in both theoretical and experimental parts. We asked for and got one year extension. That year was not free of pandemic neither. The production of wood arch elements for the experiment was very slow. Our laboratory technician retired and no one was hired instead of him. The test ring making was also delayed. Finally, we could finish the experiments but the data processing of the measurements is even now in progress.

The use the non-linear finite element software Athena was also delayed by more than half year by unexpected operational problem, the software shut down while running. First hardware problem was expected by both computer technicians and the software producer. But after fixing the hardware according to the suggestions the problem remained. After more than half year the solution was not to use the graphic card for calculation although the software producer suggests that.

So finally all the planned experiments were finished but the data processing needed to prepare papers to publish has not finished yet. Some of these data are needed to complete some theoretical parts. At least three more papers are under preparation.

3. Changes in researchers

Dr. Kachachian, Mansur and Victor Hliva joined to the research group to help us to conduct the experiments on masonry units and the arches. They operate the loading machine and the displacement measurements by electric and digital way.

4. Summary

Technically all the planned work was done, more or less with the expected results. However, there are at least three papers under preparation which must be published to make public all the results of ours.