Research report
Interacting particle systems and non-intersecting paths

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In this report, I summarize the research projects which I was working on in the period between 1 Sep 2017 and 31 Aug 2020 supported by the Postdoctoral Excellence Program of the NKFI. I wrote three papers [TV20, FV19, FV20] during the fellowship. I gave a seminar talk (Probability Seminar, Eötvos University, Budapest, 2019) and a presentation at a conference (Bernoulli–IMS One World Symposium, online, 2020) in this period. In the detailed report below I follow the numbering of problems used in the research plan and working plan submitted upon application to the Postdoctoral Fellowship.

1 KPZ equation and directed polymer models

In this research with Zsófia Talyigás (BME, Bath), we consider two directed polymer models in the Kardar–Parisi–Zhang (KPZ) universality class. They are the O'Connell–Yor semi-discrete directed polymer with boundary sources and the continuum directed random polymer. These polymer models were used in [BCFV15] to approach the solution of the KPZ equation with stationary initial condition by considering its Hopf–Cole solution, i.e. by using the Feynmann–Kac representation of the solution as the partition function of the continuum polymer model. In this work, we introduce and use a different initial condition for the KPZ equation that we call the (m,n)-spiked initial condition as a generalization of the m-spiked initial condition known in the literature.

In order to study the solution of the KPZ equation under the (m,n)-spiked initial condition, we use the corresponding (m,n)-spiked boundary perturbations for the continuum directed polymer. These boundary perturbations are constructed using two semi-discrete polymer models with independent bulk randomness and coupled boundary sources. We prove that the limiting fluctuations of the free energies rescaled by the 1/3rd power of time in both polymer models converge to the Borodin–Péché type deformations of the GUE Tracy–Widom distribution.

The paper about the results above has been published in [TV20].

2 Interacting particle systems

I did not have time to work on these topics and to obtain any new result.

3 Non-intersecting paths and tiling problems

In this research with Patrik Ferrari (Bonn), we consider uniform random domino tilings of the restricted Aztec diamond domain which is obtained by cutting off an upper triangular part of the Aztec diamond by a horizontal line. The limiting fluctuations for the boundary of the north polar region in the unrestricted model were described using the evolution of the top curve in the corresponding non-intersecting line ensemble. We represent the correlation kernel of the non-intersecting line ensemble in terms of a random walk in a novel way.

The line of restriction in the Aztec diamond is chosen so that it asymptotically touches the arctic circle (that is the limit shape of the north polar region in the unrestricted model) and it has a non-trivial interaction with the boundary of the north polar region on its fluctuation scale. We prove that the rescaled boundary of the north polar region in the restricted domain converges to the Airy_2 process conditioned to stay below a parabola as the size of the domain tends to infinity. We show the convergence of the continuum statistics and convergence of finite dimensional distributions. The limit is the top line of the hard-edge tacnode process for Brownian motion obtained for non-intersecting Brownian bridges conditioned to stay below a threshold in [FV17]. It is a one-parameter family of processes which depends on the tuning of the threshold position on the natural fluctuation scale.

The paper about these results [FV19] has been accepted for publication.

4 Scaling limits of random matrix models

In a research project in collaboration with Diane Holcomb (Stockholm), Gaultier Lambert (Zürich), Elliot Paquette (Ohio) and Bálint Virág (Toronto), we have been working on a problem closely related to the proposed one. We studied linear statistics for the Sine_β process which appears as the local bulk limit of β-ensembles. We tried to show a functional central limit theorem for the Sine_β process, that is for appropriate test functions with compact support the centered linear statistic evaluated on the points of the rescaled Sine_β process converges to a normal random variable. The idea of proof employed the counting function description of the Sine_β process introduced in [VV09] and it was using a martingale characterization of the linear statistic and the central limit theorem for martingales. Despite of promising partial results we could not complete the proof because the quadratic variation of the martingale that appeared could not be bounded as we expected. In the meantime two other papers [La19, Le19] concluded the same central limit theorems using different methods.

5 Tail decay of KPZ universality class models with random initial conditions

This research problem was not mentioned in the original research plan. With Patrik Ferrari, we consider the limiting distribution of KPZ growth models with random but not stationary initial conditions introduced in [CFS18]. The one-point distribution of the limit is given in terms of a variational problem involving a Brownian motion and an independent Airy_2 process minus a parabola. We study these two components in detail. The supremum of Brownian motion minus a parabola was already analyzed in the literature and we rely on existing results on its distribution.

The other component is the Airy_2 process minus a parabola where the coefficient of the parabola is a parameter in $(0, 1]$. The distribution of the supremum was known only in the case of coefficient 1 when it is equal to the GOE Tracy–Widom distribution. In the general case, we give upper and lower bounds on the right tail for the distribution of the supremum which match up to a logarithmic factor. These bounds follows by a directly study of the Airy_2 process and they can be of independent interest. We use these bounds to deduce the right tail asymptotic of the limiting distribution function of KPZ class models. This gives a rigorous proof and extends the results obtained by Meerson and Schmidt in [MS17].

These results are written in the preprint [FV20].

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