Final Report
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Game Theory: Concepts, methods, applications

We have investigated various topics in game theory. The motivation for these basically theoretical investigations came either from the theory itself (modelling situations with incomplete information or infinite many players, studying old solution concepts (core, Shapley value, nucleolus) and proposing new ones (soft correlated equilibrium), or from exploring the applicability of the theory (to cost / benefit allocation on networks, or to oligopolistic matching markets). Both the strategic and the cooperative approaches are represented among the questions which range from the very fundamental to the quite applied. During the alltogether six and a half years of the project we have also addressed some new questions originally not foreseen or planned.

In the proposal we stated that for the planned 48 months of the project "our goal is to produce a total of 8-12 articles in top quality periodicals of the field, and to have a total of 14-18 presentations at respected conferences". Because of the pandemic, the duration of the project was considerably extended to 80 months, so the proportionally increased target figures are cca. 14-20 articles and 23-30 conferences presentations. Despite the fact that two colleagues have become inactive during the project (retirement, maternity leave), we have succeeded to surpass all these adjusted figures: 23 articles have appeared in well-respected international journals with Impact Factor; 7 more articles are published in domestic periodicals; 2 papers have appeared in edited volumes; and we have delivered more than 25 presentations at international conferences and workshops, and more than 10 talks at domestic conferences.

The Corvinus Game Theory Seminar has continued to provide a regular forum for colleagues and students interested in game theory and its applications to economics and other social sciences. During the pandemic we went purely online, then we turned to a hybrid format. We have also organized several workshops composed of three talks related to a specific topic, where the highlighted talk have been delivered by a prominent visiting international speaker.

Following the structure of the project proposal, we summarize the achieved results topic by topic.

Soft correlated equilibrium

Forgó (2017a), and subsequently Forgó (2020) determines tight bounds for enforcement values of soft correlated equilibrium for generalized *n*-person chicken and prisoner's dilemma games, which are social dilemma games where the players choose between a high- and a low-risk action. The best outcome for a player is when she takes high risk while all the others take low risk. The socially optimal profile is, however, when everybody goes for low risk. Forgó proves that the exact enforcement value is 2 for this special class of mixed two-facility simple linear congestion games. Forgó also obtains a better bound of 3/2 for 2- and 3-person chicken games.

Forgó (2019a) gives new better bounds for the enforcement value of the soft correlated equilibrium in two-facility, simple, linear congestion games. The lower bound is 1,125. The new upper bound 1,265625 is significantly better than the already known 1,333.

Nash equilibrium, Nash bargaining solution

Forgó (2017b) demonstrates how the concept of the Limit-Nash bargaining solution as defined by Forgó and Szidarovszky (2003) can be carried over to the non-symmetric case. To this end the non-symmetric Limit-Nash bargaining solution is defined. It is found that externally given weights of the players and the relative magnitude of penalties for not being able to come to an agreement merge together in the Limit-Nash bargaining solution.

Forgó (2018), and subsequently Abaffy and Forgó (2020) study the computation of Nash equilibria of bimatrix games and identified polynomially solvable cases with special attention to symmetric random games. Based on a sample of 500 randomly generated symmetric games with matrix size 12 and 15, these authors conjecture that for finding a symmetric Nash equilibrium point it is enough to check supports up to size 4 whereas for non-symmetric and all Nash equilibrium points this number is 3 and 2, respectively.

Forgó and Komlósi (2020) address the problem of numerically determining a Nash equilibrium of a bimatrix game. It is well known that this problem is very hard in general. Identifying special cases which are solvable in polynomial time is of significance both theoretically and computationally. They define a new polynomially solvable subclass of bimatrix games via a slight generalization of negative definite matrices called ,,almost negative definite" matrices. A necessary and a sufficient condition is derived for the characterization of almost negative definite matrices.

Forgó (2019b), and subsequently Forgó and Kánnai (2020) give necessary conditions for the existence of pure Nash equilibria for concave non-cooperative games in general and then applies them to Cournot oligopoly games. If for a specified class of games there always exists a pure Nash equilibrium, then cost functions of the firms must be convex. Analogously, if for another specified class of games there always exists a pure Nash equilibrium, then revenue functions of the firms must be concave in their respective variables.

Games with incomplete information or with infinite many players

Pintér and Németh (2017) consider the weak sequential core of a transferable utility game with uncertainty. They give a necessary and sufficient condition for the non-emptiness of the weak sequential core. Furthermore, they introduce the class of generalized balanced games with universal veto control – which class properly includes the class of convex transferable utility games with uncertainty – and show that every generalized balanced game with universal veto control has a non-empty weak sequential core.

Pintér (2017) considers a strong duality theorem for infinite linear programs, formulates and proves the infinite dimensional version of the Farkas lemma, and by this result he proves a strong duality theorem for infinite linear programs.

Hellman and Pintér (2022) show that the equivalence of common priors and absence of agreeable bets of the famous no betting theorem can be generalised to any infinite space (not only compact spaces) if they expand the set of priors to include probability charges as priors. Going beyond the strict prior/no common prior dichotomy, they further uncover a fine-grained decomposition of the class of type spaces into a continuum of subclasses in each of which an epistemic condition approximating common priors is equivalent to a behavioural condition limiting acceptable bets.

Bartl and Pintér (2023) consider transferable utility cooperative games with infinitely many players and show that provided a game is bounded below, its core (in the space of bounded additive set functions) is non-empty if and only if the game is balanced. This generalizes Schmeidler’s (1967) result, where the game is assumed to be non-negative. Bartl and Pintér also generalize Schmeidler's (1967) result to the case of restricted cooperation.

Computation of weighted nucleoli in balanced games

Solymosi (2019a) considers linearly weighted versions of the least core and the (pre)nucleolus and investigates the reduction possibilities in their computation. He proves that if the core of the game is not empty, all dually inessential coalitions can be ignored when we compute the per-capita least core and the per-capita (pre)nucleolus from the dual game. This could lead to the design of polynomial time algorithms for the per-capita least core and (pre)nucleolus in specific classes of balanced games with polynomial many dually essential coalitions.

Games modeling network-restricted cooperation

Radványi (2018) considers the problem of sharing the cost of emission in supply chains and gives a characterization of the class of the induced transferable utility cooperative games, called upstream responsibility games. She also provides two characterizations of the Shapley value for upstream responsibility games by extending Dubey's and Moulin and Shenker's results on the axioms characterizing the Shapley value for the class of airport games. Based on these results, Pintér and Radványi (2019a) introduce some pollution related properties and connect them to axioms used in the cooperative game theory literature. Subsequently, Pintér and Radványi (2019b) generalize the formal notion of upstream responsibility games to a non-tree model, and provide both a primal and a dual characterizations of the class of these games. They also provide axiomatizations of the Shapley value under both characterizations.

Atay and Solymosi (2018) study a special three-sided matching game, the so-called supplier-firm-buyer game, in which buyers (customers) and sellers (suppliers) trade indirectly through middlemen (firms). They show that for these games the core coincides with the classical bargaining set and also with the Mas-Colell bargaining set.

Csercsik, Hubert, Sziklai and Kóczy (2019) propose a method to compute payoffs, reflecting the power structure, for a pipeline system with regulated third party access (which obliges the owner of a pipeline to transport gas for others, provided they pay a regulated transport fee). The method is based on an iterative flow mechanism to determine gas flows and transport fees for individual players and uses the recursive core and the minimal claim function to convert the partition function form game back into a characteristic function form game, which can be solved by standard methods. The approach is illustrated with a simple stylized numerical example of the gas network in Central Eastern Europe with a focus on Ukraine's power index as a major transit country.

Assignment markets and games

Solymosi (2023) proves that an assignment game admits a population monotonic allocation scheme (PMAS) if and only if it contains a veto player or a dominant veto mixed pair or is composed of from these two types of special assignment games. It is also shown that in a PMAS-admissible assignment game all core allocations can be extended to a PMAS, and the nucleolus coincides with the tau-value.

Solymosi (2023) investigates the sensitivity of the buyer-optimal and the seller-optimal core allocations, and that of their average, called the 'fair' allocation, in assignment markets. It is shown that if an agent unilaterally falsifies all of his stated valuations by the same amount, the agent's payoff in the 'fair' allocation cannot decrease, but can increase by at most half of that amount.

Atay, Bahel and Solymosi (2023) investigate an extension of assignment markets, where a buyer–seller pair may either trade directly or use the services of a middleman who may serve multiple buyer–seller pairs. They show that matching markets with middlemen are totally balanced: in particular, there exists a buyer-optimal (and a seller-optimal) core allocation where each buyer (seller) receives her marginal contribution to the grand coalition. In general, the core does not exhibit a middleman-optimal allocation, not even when there are only two buyers and two sellers. However, they prove that in these small markets the maximum core payoff to each middleman is her marginal contribution. Finally, they establish the coincidence between the core and the set of competitive equilibrium payoff vectors.

Applications of cooperative games to cost / benefit allocation

Balog, Bátyi, Csóka and Pintér (2017) investigate risk capital allocation methods. They analyze seven methods widely discussed in the literature or used in practice in terms of ten reasonable fairness properties. They provide proofs or counterexamples for each method and the ten properties considered. They also computed how often on average Core Compatibility is satisfied in randomly

generated risk capital allocation situations up to nine subunits in 24 treatments for all methods that do not satisfy Core Compatibility.

Csóka, Illés and Solymosi (2022) study the Shapley allocation in liability problems, when the asset value of an insolvent firm must be distributed among the creditors and the firm itself, and the firm has some freedom in negotiating with the creditors. They establish three main monotonicity properties of the Shapley allocation. First, creditors can only benefit from the increase in their claims or of the asset value. Second, the firm can only benefit from the increase of a claim but can end up with more or with less if the asset value increases, depending on the configuration of small and large liabilities. Third, creditors with larger claims benefit more from the increase of the asset value.

Oligopoly games

Bakó and Kálecz-Simon (2017a) investigate the effect of reference prices on oligopolistic firms' behavior by analizing pricing strategies and equilibrium outcomes when differentiated firms compete in Bertrand fashion and reference prices act as anchors. They show that anchoring may lower the prices charged in the long run. Furthermore, they claim that the existence of anchoring makes collusion less stable.

Bakó and Kálecz-Simon (2017b) compare quota bonuses to profit-based evaluation and sales (quantity) bonuses in a duopoly setting with independent demand shocks. Contrary to the previous findings, the authors show that under certain circumstances, either quota bonuses or sales bonuses may be optimal compensation plans.

Product differentiation, price discrimination, misinformation in markets

Bakó, Neszveda and Dezső (2018) investigate some implications of recent results about salience on loan decisions. Using the framework of focus-weighted utility they show that consumers might take out loans even when that yield them negative utility due to the focusing bias. They suggest, however, that this can be counterbalanced and consumers might be more prudent in their decisions and less likely to take out such loans when the usual fixed-installments plan is coupled with an equivalent decreasing-installments option. Moreover, this is true even for loans with prepayment options or when borrowers take default into consideration.

Bakó and Fátay (2019) analyze asymmetric two-sided markets. Two types of agents are assumed to interact with each other and agents of one type derive utility from inter-group interactions, while the other type of agents benefit from intra-group rather than from inter-group interactions as it is assumed in the standard symmetric two-sided markets model. The results suggest that a monopoly platform with asymmetric externalities might charge a significantly higher price than a platform where the external effects are symmetric. Competition forces asymmetric platforms only to charge smaller prices on the side that serves as a loss leader, but the pricing strategy on the other side where the monopoly power is being exploited is not affected. Moreover, the effect of competition is further restricted when multi-homing is not available for the consumers.

Bakó, Berezvai, Isztin and Vigh (2020a, 2020b) in two subsequents papers analyze the effect of price differentiation on bicycle sharing platforms. They exploit a natural experiment provided by the forced exit of Uber from Budapest to assess the effect of Uber on bicycle-sharing system ridership. The authors find that banning Uber caused a significant decrease in usage among frequent users especially on weekdays, suggesting a complementary relationship between these services. On the other hand, their findings indicate that ad hoc users mainly use bicycle-sharing system and Uber as substitutes.

Bakó and Neszveda (2020) investigate salience theory for multi-dimensional decisions which has been successfully used to explain a wide range of empirical and experimental phenomena. They show that salience theory carries a notable flaw inasmuch under certain circumstances it suggests

that a salient thinker may prefer a dominated option even when a strictly dominant alternative is available. To solve this problem the authors propose a possible alteration of salience theory.

Bakó and Horváth (2020) study the pricing equilibrium of asymmetric two-sided markets. They explore the pricing policy of the platforms, and analyze the incentives of the firms to customize their products and services to the users. The results suggest that profit-maximizing platforms will provide users with maximum customization at a positive price, or a limited customization at zero price. They also show that platforms with monopoly power on the users' side might not be able to take advantage of their market power if they face competition on the advertisers' side of the market.

Bakó and Isztin (2022) consider information avoidance in product markets. They show that misinformation can be an equilibrium outcome if consumers receive disutility when proven wrong in their product quality assessment. Consumers, however, are assumed to respond to market and other incentives. The incentive to learn contradicting information increases in the price of the product. The possibility of false information in equilibrium provides a rationale for regulation or establishing tort liability. However, even though regulation dampens the effects of information aversion, laissez-faire still might be better for consumers even when regulation is highly effective.

Kiss, Kóczy, Pintér and Sziklai (2022), in a laboratory experiment, investigate whether sorting according to risk attitudes has any effect on overpricing on asset markets. The research is motivated by the fact that filtering out risk-averse investors is practiced widely in Europe and is in line with the Markets in Financial Instruments Directive of the EU. They show that risk sorting can explain overpricing only partially: markets with the most risk-tolerant traders exhibit larger overpricing than markets with the most risk-averse traders. In their study, risk aversion does not correlate with gender or cognitive abilities, bringing in an additional factor to understand overpricing.

Identification and ranking of influential members of a group

Sziklai (2018) uses the group identification framework to design an expert selection mechanism. He establishes an axiomatization to show that the algorithm is theoretically well-founded. Furthermore he presents a case study using citation data to demonstrate its effectiveness, and compares its performance with classical centrality measures.

Sziklai (2021) presents a novel algorithm to rank smaller academic entities such as university departments or research groups within a research discipline. The Weighted Top Candidate algorithm is a generalisation of an expert identification method. The axiomatic characterisation of the Weighted Top Candidate shows why it is especially suitable for scientometric purposes.

Other papers and works

Radványi (2019) provides an introductory overview on how cooperative game theoretic solution concepts can be used for a wide range of practical cost allocation problems with an underlying fixed tree network structure where nodes of the network denote users using a service via the network. The aim is to find ”fair” allocations for the costs arising from using the system.

Solymosi (2019b) provides an introductory overview on the coincidence of the core with the best known bargaining sets, namely the classical, the reactive, the semireactive, and the Mas-Colell bargaining sets. The review of known results is augmented by some new observations and open questions.

Sziklai (jointly with Jochen Staudacher, Károly Héberger, and Attila Gere) developed the R package rSRD (sum of ranking differences statistical test) that provides an user friendly platform for the novel statistical method Sum of Ranking Differences designed to compare solutions based on a reference point. The software package has been accepted in 2022 to be made publicly available in the CRAN (The Comprehensive R Archive Network) repository at https://cran.r-project.org/ .

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