Maldacena's famous AdS/CFT correspondence is a holographic duality conjecture, which states that type-II-B superstring-theory defined on the product of a 5-dimensional anti-de Sitter space (AdS_5) and a 5-dimensional sphere (S^5), is equivalent to the 4-dimensional maximally supersymmetric Yang-Mills theory. The duality is of weak/strong type, which means that weakly coupled gauge theory is mapped to strongly coupled string theory and vice versa. This implies that the duality conjecture can be proven and solved only by nonperturbative methods. In the planar limit, the integrability discovered on both sides of the correspondence allows us to treat the problem with exact methods. The main goal of our project was to compute the most important physical quantities of the supersymmetric Yang-Mills theory through the solution of the dual string theory. More concretely we wanted to determine the spectrum, form-factors, and n-point correlation functions in the string theory.

The first part of the research focused on the investigation of the spectrum. We used both analytical and numerical methods to reach our goals.

We started our research by the study of boundary problems. Namely, we investigated the ground state energy of a pair of open strings stretching between a coincident D3- anti-D3-brane system in $AdS_5 \times S^5$ superstring-theory. The main motivation for this study was that according to string theory predictions the ground state of such a system should be tachyonic for large values of the 't Hooft coupling. We analyzed our D-brane anti-D-brane system by the following techniques: perturbation theory in the super Yang-Mills theory (SYM), boundary Lüscher formula and the thermodynamic Bethe Ansatz (TBA) integral equations [1]. We found agreement between a perturbative high order diagrammatic calculation in the super Yang-Mills theory and the leading order boundary Lüscher correction. We derived and solved numerically the TBA equations for the ground state of the model. We experienced that beyond a certain value of the coupling constant, the iterative numerical method for solving the TBA equations fails to converge anymore. We showed analytically, that beyond this critical point the usual TBA description breaks down. Since at this critical point the energy of the open string becomes zero, we interpreted this point as a turning point where the ground state energy becomes imaginary and the string state becomes tachyonic. In order to get a deeper insight into the breakdown of the usual TBA method, we transformed the infinite component TBA equations into finite component hybrid-NLIE equations [2] and performed their numerical solution. Straightforward numerical methods failed to converge, this is why we worked out new numerical methods to solve the equations. The numerical method which proved to be the most efficient is as follows: after the appropriate discretization of the nonlinear integral equations, the discretized equations are considered as a set of nonlinear algebraic equations and they are solved by the Newton-method. We also discussed how the TBA description could be extended beyond the critical point. We found that if the extended equations have a solution, then the energy must be imaginary in accordance with the tachyonic expectations.

Next, corresponding to the research plan, by applying the hybrid-NLIE method, we started the numerical investigation of the spectrum of anomalous dimensions in the sl(2) sector of the super Yang-Mills theory. In the meantime Gromov el. al. [A] published a new and very effective numerical method for solving numerically the spectral problem of the theory. In this approach the solutions of a set of nonlinear Riemann-Hilbert equations, the so-called Quantum Spectral Curve (QSC) equations determine the coupling constant dependence of the spectrum. Since this new method makes it possible to reach higher numerical precision, than our hybrid-NLIE method, we started to apply this method for our further numerical investigations.

In [3] we solved numerically the Quantum Spectral Curve equations corresponding to some twist-2 single trace operators with even spin to determine their anomalous dimensions in the strong coupling regime. In the studied S = 2, 4, 6, 8 cases, our numerical results confirmed the analytical results, conjectured previously in the literature for the first 4 coefficients of the strong coupling expansion for the anomalous dimensions of twist-2 operators. In the case of the Konishi operator (S=2), due to the high precision of the numerical data, we could give numerical predictions to the values of two further coefficients, as well. By the confirmation of the 4 leading coefficients of the strong coupling expansion, we also verified that the string states corresponding to our gauge theory operators are spinning folded strings with appropriate quantum numbers. We note that with the help of our numerical method, we could determine the anomalous dimensions under investigation with more that 20-digits of precision. This is probably the highest precision ever reached in theoretical calculations of physical quantities in a non-abelian gauge theory.

We also investigated the strong coupling behavior of the coefficients in the power series representation of the unknown-functions. Based on our numerical data, in the regime, where the index of the coefficients is much smaller than the fourth root of the 't Hooft coupling, we conjectured that the coefficients have polynomial index dependence at strong coupling. This allowed us to propose a strong coupling series representation for the unknown-functions being valid far enough from the real short cut.

In the next step we started the numerical computation of the generalized quark-antiquark potential as functions of the coupling constant and the “theta-angle.” We investigated a lot of effort into the project, we programmed the Quantum Spectral Curve equations corresponding to the quark-anti-quark potential and started to solve the equations numerically. The starting point of the numerical iterations was the analytical perturbative solution of the equations. After some initial success we faced with serious technical difficulties in the numerical solution. First it turned out that the computations require very more that 100-digits numerical precision within the numerical computations. Second the iterations slowed down as we increased the coupling constant, such that we could increase the value of the coupling constant in very small steps. These two difficulties made the computations so time consuming that to reach the required strong coupling regime within reasonable time seemed to be impossible. This is why we decided to finish the computations.

The second part of the project focused on the form-factors. In the planar limit the string theory reduces to a 2-dimensional integrable interacting quantum field theory, the so-called string sigma-model. Since in the planar limit, the geometry of the worldsheet is that of a cylinder, the model has to be solved in finite volume. To determine the finite volume form factors proves to be an extremely difficult task in the string sigma model of our interest. This is why to get a deeper understanding of the finite volume form factors in integrable quantum field theories, we considered the sine-Gordon model, as a toy model and started to investigate its finite volume form-factors. Using an integrable lattice regularization of the model, we computed the finite volume diagonal form-factors of the U(1) current [4] and the trace of the stress-energy tensor [5] of the model. These diagonal form-factors (or in other words expectation values) can be obtained by solving a special set of linear integral equations. The kernels of the equations are related to the solution of the nonlinear integral equations, which describe the state in which the expectation value is taken. In the large volume limit, we solved the linear integral equations governing the finite volume diagonal form-factors. As a result we conjectured a large volume series representation for the finite volume expectation values of local operators in pure soliton states. The remarkable property of this representation is that it contains the “connected” infinite volume diagonal form factors and the solution of the nonlinear integral equations governing the energy of the sandwiching state.

Publications containing the scientific achievements of the report:

[1.] Z. Bajnok, N. Drukker,Á. Hegedűs, R.I. Nepomechie, L. Palla, C. Sieg, R. Suzuki:
“The spectrum of tachyons in AdS/CFT”
JHEP 1403 (2014) 055
arXiv:1312.3900 [hep-th]

[2.] Árpád Hegedűs: Extensive numerical study of a D-brane, anti-D-brane system in AdS5 /CFT4
JHEP 1504 (2015) 107
arXiv:1501.07412 [hep-th]

[3.] Á. Hegedűs, J. Konczer:
“Strong coupling results in the AdS5/CFT4 correspondence from the numerical solution of the quantum spectral curve equations”
JHEP08(2016)061 ,
arXiv:1604.02346 arXiv:1312.3900 [hep-th]

[4.] Á. Hegedűs:
“Lattice approach to finite volume form-factors of the Massive Thirring (Sine-Gordon) model,”
JHEP08(2017)059 ,
arXiv:1705.00319

[5.] Á. Hegedűs:
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arXiv:1710.09583 [hep-th]

Other references:

[A.] N. Gromov,F. Levkovich-Maslyuk, G. Sizov: „Quantum spectral curve and the numerical solution of the spectral problem in ADS5/CFT4”
JHEP06 (2016) 036
arXiv:15.04.06640 [hep-th]