Equilibria and transient dynamics of non-smooth mechanical systems

Final research report

1. Introduction

The aim of the project was to improve our knowledge on the existence, the stability, and the attractivity of equilibria in mechanical systems with contacts as well as to explore their dynamics in a small neighborhood of equilibria. Rigid bodies and multibody systems were in the main focus of the project, nevertheless we proposed to extend the investigations to elastic rods in the last phase of the research. The project aimed to explore key characteristics of complex system dynamics in the presence of contacts (e.g. robust convergence to equilibrium), and to analyze and resolve the inconsistency and indeterminacy of solutions occurring in models combining contact with rigid body theory.

Five key questions have been identified in the research proposal. One of them was a fundamental theoretical question addressing severe limitations of rigid body framework, whereas the others referred to complex behavior produced by simple models, which are important for engineering applications. The results of the project are summarized for each question below.

2. Statistics of stable postures

We proposed to study *landing probabilities*, i.e. the probability that a given rigid object with multiple stable orientations on a horizontal plane converges to a given one, if it is dropped with random initial conditions. In order to achieve this goal, a large number of random polyhedra have been generated and their dynamics from random initial conditions have been simulated by a custom-made program code. A large dataset of simulated landing probabilities has been created. The simulation results were verified against experimental results from the literature. Existing simple estimators of landing probabilities available in the literature have been evaluated against this dataset, and several new estimators have been proposed. We were able to develop an estimator with average estimation error that was less than half of the error of the best one previously available. The main results were published in a leading journal of IEEE [1]. In addition, algorithmic questions of new estimation methods were presented at conferences with a follow-up journal publication [2].

We also proposed to study the possibilities of manipulating landing probabilities (instead of merely estimating them). Landing probabilities play key role in automated part feeding, where the goal of engineers is to design a mechanical environment in which processed parts rotate into one single orientation with high probability (or ideally they have a single stable orientation). Motivated by earlier attempts to develop "universal part feeders" in two dimensions that satisfy this requirement for any part, we made initial steps towards the development of a feeder in which any three dimensional object rotates into a unique orientation. A theoretical framework has been worked out, with a detailed discussion of issues of practical implementation. Our theoretical ideas were verified by physical experiments. The results have been published [4] in a recently launched high-quality journal of IEEE with impact factor to be announced first in 2018[1].

3. Transient dynamics and impact sequences

We proposed deeper, analytical and numerical investigation of the transient dynamics of an object dropped onto a flat plane with multiple points reaching the plane almost simultaneously. In those cases when the effect of external forces is negligible, we were able to explain why the dynamics is often insensitive to the exact order of impact locations. Our main tool was the application and the extension of the theory of common invariant cones of linear operators. In a detailed study of objects with rotational symmetries, we have found a sharp boundary in the space of model parameters, on one side of which "clattering" may occur, i.e. the object undergoes a rapid, infinite sequence of impacts terminating at the state of complete immobility. A journal paper summarizing our results is currently under review and its full text is available[2].

We found a unique application to our theory when the landing of spacecraft Philae on the surface of comet 67P/Churyumov–Gerasimenko on 12 November, 2014 took an unexpected turn. Due to failure of several subsystems designed to attach the lander of the comet upon touchdown, the landing of the three-legged spacecraft included a short episode with several impacts, followed by several hours of uncontrolled motion in the micro-gravitational environment. The final landing position of the lander remained unknown for 2 years[3]. In collaboration with A. Balázs from Wigner Research Centre for Physics, Budapest, we obtained and analysed data series of in situ measurements of the electric power provided by the solar panels. These data were used to partially reconstruct the landing and the rotational motion of the lander, which yielded useful implications about its fate. Our results have been published in [7].

Our theoretical investigations were also extended to systems subject to gravitational forces. We analysed the effect of impacts on the motion of a spinning disk (a.k.a. Euler's disk), which is a popular scientific model problem. It exhibits counterintuitive behaviour and a singularity: energy absorption appears to induce gradually accelerating motion accompanied by a loud noise of increasing frequency, until motion abruptly terminates. Many authors have studied the physical mechanisms behind these phenomena. We proposed for the first time, that rapid sequences of impacts (due to imperfect object shapes or elastic vibrations) may play an important role here. Our detailed analysis demonstrated the existence of ranges in the space

[1] http://www.ieee-ras.org/publications/ra-l/information-for-authors-2#ten
[2] full text available at https://arxiv.org/abs/1704.03044
[3] http://www.esa.int/Our_Activities/Space_Science/Rosetta/Philae_found

of model parameters, where the effect of impact sequences becomes dominant over any other energy absorption mechanism. These results have been published in [9].

Finally, we also proposed the investigation of impact sequences from the point of view of shock protection. We studied the motion of a damped harmonic oscillator (representing a sensitive part in an electric device) attached to a rigid rod dropped to a rigid plane (representing the rest of the device) by numerical simulation. We decided to measure the "shock" experienced by the investigated part as $S=A/A_0$ where A is the maximum amplitude of the oscillator during motion and A_0 is the maximum amplitude if the device instantly stops instead of undergoing a sequence of impacts. We were able to demonstrate the important role of appropriate positioning of sensitive parts within the device as well as sensitivity to the mass distribution of the device. Most importantly, parts near the centre of mass exhibit smaller shocks in most cases, and a small radius of gyration of the device appears to be advantageous for shock protection. Our results offer new methodology in the design of robust, shock-resistant electronic devices. A paper presenting these results is in preparation.

4. Painlevé paradoxes

We proposed to study Painlevé's non-uniqueness and non-existence paradoxes. During the research project we undertook a detailed investigations of planar systems with 2 point contacts generalizing existing results for the case of 1 single contact. A new classification scheme has been developed. By using contact regularization techniques, previously unknown forms of contact dynamics, new cases of solution non-uniqueness, and non-existence have been uncovered. Several myths based on the analysis of simple model problems have been refuted. The results have been published in [5].

The case of one single point contact has also been revisited. In collaboration with Alan R. Champneys from Bristol University, we proposed a uniform approach to contact problems in 2 or 3 dimensions. We pointed out some fundamental differences between the case of 3D motion, and previously studied 2D problems. This work has been published in [6].

Finally, we also considered the equilibria of systems with arbitrary number of point contacts. A new stability theorem has been proved, which demonstrates that a large class of equilibria (including those subject to solution indeterminacy) are stable against small perturbations. We also demonstrated the applicability of this result in robotic grasping and locomotion. The results have been published in [3].

5. Dynamic stability of rigid bodies

We proposed to develop new stability conditions of rigid bodies with two point contacts in two dimensions. This is an important model problem, and an idealized representation of a quasi-static bipedal robot in two dimensions. During the project, we developed a stability test based on the method of Poincaré maps to provide a nearly exact characterization of Lyapunov-stable postures when the contacts are ideally inelastic. Remaining tasks include the adaptation of our solution method to some special configurations, including ones with Painlevé's non-uniqueness paradox. We demonstrated semi-analytically that stability may depend on model parameters in highly nonlinear and counter-intuitive way. The loss of Lyapunov stability was also successfully demonstrated by physical experiments, and a large dataset about the effect of model parameters on stability has been acquired by numerical simulation. Our results have been published in [8].

6. Buckling under friction

We proposed to study the motion of elastica, which expand while their motion is resisted by friction. We have demonstrated that these problems are out of the scope of classical stability theory, since they cannot be formulated as fast, autonomous dynamics coupled with slow variation of a bifurcation parameter. Instead of that, they are formulated as non-autonomous dynamical systems without time scale separation. Such systems often show initial convergence towards a trivial solution, which is later replaced by divergence (Fig 1a). Capturing the critical point between these two types of behavior requires an appropriate notion of stability against infinitesimal perturbations, which is defined for motion over finite time intervals. We successfully created such a definition and determined the critical point associated with the loss of stability semi-analytically. We pointed out fundamental qualitative differences between this behavior and classical forms of elastic buckling. In collaboration with A. Á. Sipos from BME, a numerical simulation tool has been developed by using variational methods. This was used to study the post-critical behavior of the system (Fig. 1). We found that our systems lack a well-defined post-critical shape, and the number of inflexion points grow gradually. Numerical results also suggest that the growing rod asymptotically converges to a special “figure 8” shape after long time and the distance between its endpoints converges to a finite value. These results yield important insights into limitations of designing slender, flexible manipulators and into the growth of thin roots. Experimental verification of our simulation results was completed recently by an undergraduate student. Those results will soon be submitted as a Student research project (TDK) and will also be included in a journal publication, which is currently in preparation.

7. Summary

Rigid body theory has fundamental limitations when systems have contact interaction. The present research project made significant steps towards exploring and relaxing some of these limitations.

First, our theoretical results about Painlevé’s paradox produced significant new results despite a century-long history of development and a growing interest during the past decade. We also exploited these theoretical results in order to describe the behavior of contact systems when we were looking for conditions of stability.

The second major difficulty about contact systems is complex, nonlinear, and non-smooth behavior. We used and developed several mathematical tools including Poincaré maps, the theory of invariant cones, and theory of finite-time stability, which allowed us to capture some of those features of complex system dynamics, which are important in engineering applications.

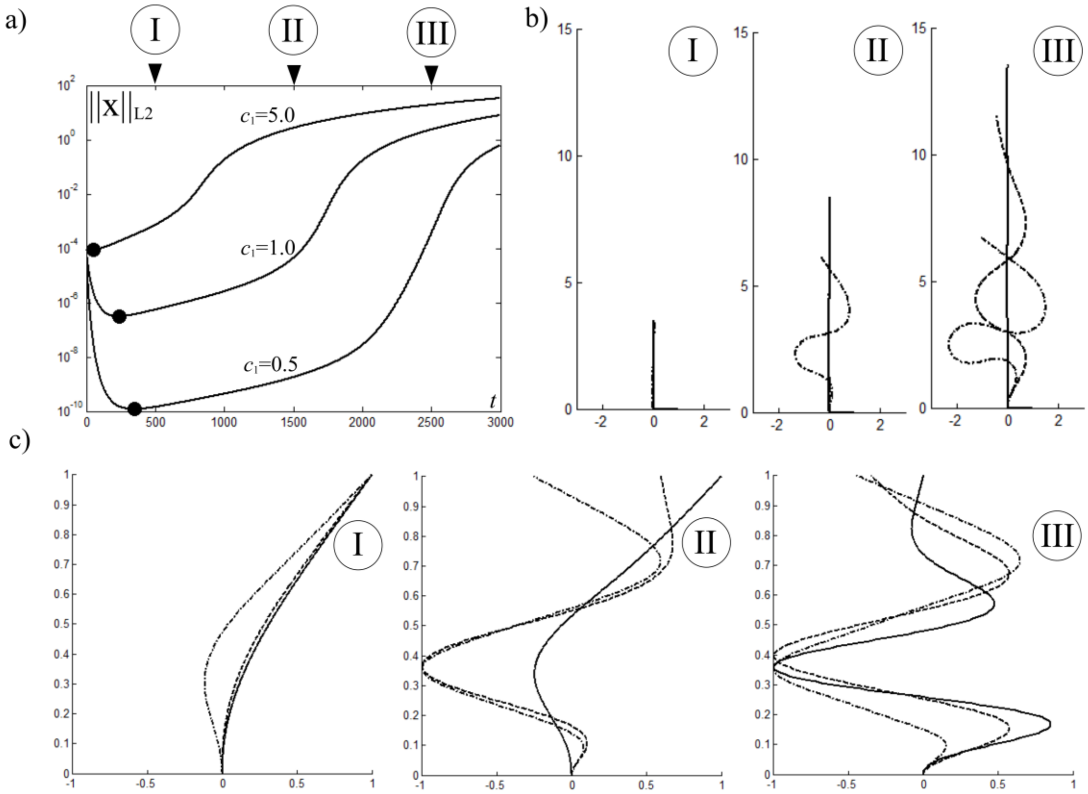

Fig. 1: example of numerical simulation of growth under friction. panel a) show deviation of solutions for 3 different values of the coefficient of friction c_1 from trivial solution as a function of time. Filled circles denote minimum points. b) show the physical shapes of growing rods at 3 time instants. Panel c) also illustrates post-critical shapes by showing magnified transversal displacements as a function of arclength.

Last but not least, contact phenomena such as impact and friction lack simple and reliable models. Consequently, the robustness of system behavior against modeling errors becomes crucial. Our work found in several cases that important properties of the dynamics are robust, despite the sensitivity and unpredictability of individual motion trajectories.

We believe that our achievements in these three important directions will provide new tools for the analysis of contact systems in the future.

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