

A short summary of the research results, listed by publications (papers are ordered corresponding to the list in the electronic system):

1. A. Besenyei, D. Petz: Partial subadditivity of entropies, *Linear Alg. Appl.* 439 (2013), 3297-3305., (2013).

In the paper some new inequalities related to the subadditivity and strong subadditivity of the standard entropy and the Tsallis entropy are established in the discrete probabilistic case. The inequalities express a kind of partial subadditivity in the sense that the classical subadditivities can be recovered from them. The matrix analogues of these inequalities are also discussed and a conjecture is formulated in this context.

2. D. Petz, G. Tóth: Extremal properties of the variance and quantum Fisher information, *Phys. Rev. A* 87, 032324., (2013).

We show that the variance is its own concave roof. For rank-2 density matrices and operators with zero diagonalelements in the eigenbasis of the density matrix, we prove analytically that the quantum Fisher information is four times the convex roof of the variance. Strong numerical evidence suggests that this statement is true even for operators with nonzero diagonal elements or density matrices with a rank larger than 2.

3. Z. Léka, D. Petz: Some decompositions of matrix variances, *Probab. Math. Statist.*, 33:(2) 191-199, (2013).

When D is a density matrix and A_1, A_2 are self-adjoint operators, then the standard variance is a 2×2 matrix. The main result in this work states the variance matrix can be decomposed as the sum of variances determined by projections such that their convex hull contains D .

4. D. Petz, D. Virostek: A characterization theorem for matrix variances, *Acta Sci. Math.* 80, 681-687, (2014).

Some recent papers formulated sufficient conditions for the decomposition of matrix variances. A statement was that if we have one or two observables, then the decomposition is possible. In this paper we consider an arbitrary finite set of observables and we present a necessary and sufficient condition for the decomposition of the matrix variances.

5. F. Hiai, D. Petz: Introduction to matrix analysis and applications, Springer „Universitytext” series, (2014).

This is a comprehensive survey book on matrix analysis and its applications.

6. P. E. Frenkel, M. Weiner: On vector configurations that can be realized in the cone of positive matrices, Linear Alg. Appl. 459, 465-474., (2014)

Let v_1, \dots, v_n be vectors in a scalar product space. Can we find a natural number d and some positive (semidefinite) complex matrices A_1, \dots, A_n of size $d \times d$ such that $\text{Tr}(A_j A_k) = \langle v_j, v_k \rangle$ for all $j, k=1, \dots, n$? Of course, for such a system of matrices to exist, all scalar product values $\langle v_j, v_k \rangle$ must be nonnegative reals. We prove that if $n \leq 5$ then this trivial necessary condition is also a sufficient one and find an appropriate example showing that from $n=6$ this is not so - even if we allow realizations by positive operators in a von Neumann algebra with a faithful normal tracial state.

7. Z. Lékai: Time regularity and functions of the Volterra operator, Stud. Math., 220:(1) 1-14, (2014).

Our aim is to prove that for any fixed $1/2 < a < 1$ there exists a Hilbert space contraction T such that $\sigma(T) = \{1\}$ and T has time regularity n^{-a} , and the estimate is sharp. This answers Zemánek's question on the time regularity property.

8. A. Besenyei, Z. Lékai: Leibniz seminorms in probability spaces, J. Math. Anal. Appl., 429:(2), 1178–1189, (2015).

In this paper we study the (strong) Leibniz property of centered moments of bounded random variables. We shall answer a question raised by M. Rieffel on the non-commutative standard deviation.

9. D. Petz, D. Viosztek: Some inequalities for quantum Tsallis entropy related to the strong subadditivity, Math. Inequal. Appl. 18, 555-568, (2015).

In this paper we investigate the inequality $S_q(\rho_{123}) + S_q(\rho_2) \leq S_q(\rho_{12}) + S_q(\rho_{23})$ (*) where ρ_{123} is a state on a finite dimensional Hilbert space $H_1 \otimes H_2 \otimes H_3$, and S_q is the Tsallis entropy. It is well-known that the strong subadditivity of the von Neumann entropy can be derived from the monotonicity of the Umegaki relative entropy. Now, we present an equivalent form of (*), which is an inequality of relative quasi-entropies. We derive an inequality of the form $S_q(\rho_{123}) + S_q(\rho_2) \leq S_q(\rho_{12}) + S_q(\rho_{23}) + f_1(\rho_{123})$, where $f_1(\rho_{123}) = 0$. Such a result can be

considered as a generalization of the strong subadditivity of the von Neumann entropy. One can see that (*) does not hold in general (a picturesque example is included in this paper), but we give a sufficient condition for this inequality, as well.

10. J. Pitrik, D. Virostek: On the joint convexity of the Bregman divergence of matrices, Lett. Math. Phys. 105:5, 675-692, (2015).

We characterize the functions for which the corresponding Bregman divergence is jointly convex on matrices. As an application of this characterization, we derive a sharp inequality for the quantum Tsallis entropy of a tripartite state, which can be considered as a generalization of the strong subadditivity of the von Neumann entropy. (In general, the strong subadditivity of the Tsallis entropy fails for quantum states, but it holds for classical states.) Furthermore, we show that the joint convexity of the Bregman divergence does not imply the monotonicity under stochastic maps, but every monotone Bregman divergence is jointly convex.

11. L. Molnár, D. Virostek: On algebraic endomorphisms of the Einstein gyrogroup, J. Math. Phys. 56, 082302, (2015).

We describe the structure of all continuous algebraic endomorphisms of the open unit ball B of R^3 equipped with the Einstein velocity addition. We show that any nonzero such transformation originates from an orthogonal linear transformation on R^3 .

12. M. Matolcsi and M. Weiner: An improvement on the Delsarte-type LP bound with application to MUBs, Open Syst. Inf. Dyn. 22, 1550001., (2015)

The linear programming (LP) bound of Delsarte can be applied to several problems in various branches of mathematics. We describe a general Fourier analytic method to get a slight improvement on this bound. We then apply our method to the problem of mutually unbiased bases (MUBs) to prove that the Fourier family $F(a; b)$ in dimension 6 cannot be extended to a full system of MUBs.

13. P. E. Frenkel and M. Weiner: Classical information storage in an n-level quantum system, Commun. Math. Phys. 340, 563-574., (2015).

A game is played by a team of two --- say Alice and Bob --- in which the value of a random variable x is revealed to Alice only, who cannot freely communicate with Bob. Instead, she is given a quantum n-level system, respectively a classical n-state system, which she can put in possession of Bob in any state she wishes. We evaluate how successfully they managed to store and recover the value of x in the used system by requiring Bob to specify a value z and giving a

reward of value $f(x,z)$ to the team. We show that whatever the probability distribution of x and the reward function f are, when using a quantum n -level system, the maximum expected reward obtainable with the best possible team strategy is equal to that obtainable with the use of a classical n -state system. The proof relies on mixed discriminants of positive matrices and --- perhaps surprisingly --- an application of the Supply--Demand Theorem for bipartite graphs.

14. Z. Lékai: A note on central moments in C^* -algebras, *J. Math. Inequal.*, 9:(1), 165–175. (2015)

We present sharp estimates on the maximum of k th central moments of normal elements in C^* -algebras. We shall obtain an estimate for the upper bound of weak moments of general elements as well.

15. D. Viosztek: Quantum f -divergence preserving maps on positive semidefinite operators acting on finite dimensional Hilbert spaces, *Linear Algebra Appl.* 501, 242–253, (2016).

We determine the structure of all bijections on the cone of positive semidefinite operators which preserve the quantum f -divergence for an arbitrary strictly convex function f defined on the positive halfline. It turns out that any such transformation is implemented by either a unitary or an antiunitary operator.

16. D. Viosztek: Maps on quantum states preserving Bregman and Jensen divergences, *Lett. Math. Phys.* 106(9), 1217-1234, (2016).

We describe the structure of the bijective transformations on the set of density operators which preserve the Bregman f -divergence for an arbitrary differentiable strictly convex function f . Furthermore, we determine the preservers of the Jensen f -divergence in the case when the generating function f belongs to a recently introduced function class called Matrix Entropy Class.

17. L. Molnár, D. Viosztek: Continuous Jordan triple endomorphisms of P_2 , *J. Math. Anal. Appl.* 438(2), 828-839, (2016).

We describe the structure of all continuous Jordan triple endomorphisms of the set P_2 of all positive definite 2×2 matrices thus completing a recent result of ours. We also mention an application concerning sorts of surjective generalized isometries on P_2 and, as second application, we complete another former result of ours on the structure of sequential endomorphisms of finite dimensional effect algebras.

18. L. Molnár, J. Pitrik, D. Viosztek: Maps on positive definite matrices preserving Bregman and Jensen divergences, *Linear Algebra Appl.* 495, 174–189, (2016).

In this paper we determine those bijective maps of the set of all positive definite $n \times n$ complex matrices which preserve a given Bregman divergence corresponding to a differentiable convex function that satisfies certain conditions. We cover the cases of the most important Bregman divergences and present the precise structure of the mentioned transformations. Similar results concerning Jensen divergences and their preservers are also given.

19. Z. Lékai: Some inequalities for central moments of matrices, *Linear Algebra Appl.*, 496, 246–261. (2016).

In this paper we study non-commutative central moment inequalities with a focus on whether the commutative bounds are tight in the non-commutative case. We prove that the answer is affirmative for the fourth central moment and several particular results are given in the general case. As an application, we shall present some lower estimates of the spread of Hermitian and normal matrices as well.

20. Z. Lékai: A note on extremal decomposition of covariances, *Rocky Mountain J. Math.*, 46 (2), 571–580. (2016).

We present an elementary approach to extremal decompositions of (quantum) covariance matrices determined by densities. We give a new proof on former results and provide a sharp estimate of the ranks of the densities that appear in the decomposition theorem.

21. T. Cooney, M. Mosonyi, M. M. Wilde: Strong converse exponents for a quantum channel discrimination problem and quantum-feedback-assisted communication; *Communications in Mathematical Physics*, Volume 344, Issue 3, pp. 797–829, (2016).

We study a hypothesis testing problem that naturally approximates between binary state discrimination and channel discrimination, namely, discriminating between a given channel and a replacer channel, that yields the same output state on every input. We determine the explicit strong converse exponent and the optimal Stein type exponent for this task, and show that adaptive strategies do not give an advantage over non-adaptive ones.

22. D. Viosztek: Applications of an intersection formula to dual cones. *Bull. Austral. Math. Soc.*, in press, (2017). Available online: <https://arxiv.org/abs/1704.00670>

We give a succinct proof of a duality theorem obtained by R\ev\esz in 1991 which concerns extremal quantities related to trigonometric polynomials. The key tool of our new proof is an intersection formula on dual cones in real Banach spaces. We show another application of this intersection formula which is related to the integral estimates of non-negative positive definite functions.

23. D. Virosztek: Connections between centrality and local monotonicity of certain functions on C^* -algebras. *J. Math. Anal. Appl.* 453, 221-226, (2017).

We introduce a quite large class of functions (including the exponential function and the power functions with exponent greater than one), and show that for any element f of this function class, a self-adjoint element a of a C^* -algebra is central if and only if $a \leq b$ implies $f(a) \leq f(b)$. That is, we characterize centrality by local monotonicity of certain functions on C^* -algebras. Numerous former results (including works of Ogasawara, Pedersen, Wu, and Moln\ar) are apparent consequences of our result.

24. F. Hiai, M. Mosonyi: Different quantum f -divergences and the reversibility of quantum operations; *Reviews in Mathematical Physics*, Vol. 29, No. 7, 1750023, (2017).

In this paper, we give a systematic overview of the various concepts of quantum f -divergences, with a main focus on their monotonicity under quantum operations, and the implications of the preservation of a quantum f -divergence by a quantum operation. In particular, we compare the standard and the maximal f -divergences regarding their ability to detect the reversibility of quantum operations. We also show that these two quantum f -divergences are strictly different for non-commuting operators unless f is a polynomial, and obtain some analogous partial results for the relation between the measured and the standard f -divergences. We also study the monotonicity of the α - z -Rényi divergences under the special class of bistochastic maps that leave one of the arguments of the Rényi divergence invariant, and determine domains of the parameters α, z where monotonicity holds, and where the preservation of the α - z -Rényi divergence implies the reversibility of the quantum operation.

25. Gy. P. Gehér, T. Titkos: A characterization of isometries with respect to the Lévy-Prokhorov metric, *Annali della Scuola Normale Superiore di Pisa - Classe di Scienze*, to appear, 2017.

In this paper we give the complete description of the structure of surjective Lévy–Prokhorov isometries on the space of all Borel probability measures on an arbitrary separable real Banach space. Our result can be considered as a generalisation of the characterisation of Lévy isometries

of the space of all probability distribution functions on the real line.

26. H.Y. Chen, Gy.P. Gehér, C.N. Liu, L. Molnár, D. Viosztek, N.C. Wong: Generalized isometries of the positive definite cone with respect to the quantum χ^2 α -divergences. Lett. Math. Phys. (2017), published online: <http://dx.doi.org/10.1007/s11005-017-0989-0>

We describe the structure of all bijective maps on the cone of positive definite operators acting on a finite and at least two-dimensional complex Hilbert space which preserve the quantum χ^2 α -divergence for some $\alpha \in [0,1]$. We prove that any such transformation is necessarily implemented by either a unitary or an antiunitary operator. Similar results concerning maps on the cone of positive semidefinite operators as well as on the set of all density operators are also derived.

27. Z. Lékai: On the Leibniz rule for random variables, to appear in Math. Inequal. Appl. (2017).

Here we prove a Leibniz-type inequality for the spread of (real-valued) random variables in terms of their L_p -norms. The result is motivated by the Kato--Ponce inequality and Rieffel's Leibniz property.

28. Z. Lékai: Symmetric seminorms and the Leibniz property, J. Math. Anal. Appl., 452:(1), 708-725. (2017).

We show that certain symmetric seminorms on \mathbb{R}^n satisfy the Leibniz inequality. As an application, we obtain that L_p norms of centered bounded real functions, defined on probability spaces, have the same property. Even though this is well-known for the standard deviation it seems that the complete result has never been established. In addition, connect the results with the differential calculus introduced by Cipriani and Sauvageot and Rieffel's non-commutative Riemann metric.

29. Z. Lékai: On discrete time regularity of bounded linear operators, to appear in Banach Center Publ. (In : Operator Theory), (2017).

This paper offers a survey on recent results concerning time regularity properties of bounded linear operators in Banach and Hilbert spaces.

30. M. Kolountzakis, M. Matolcsi, M. Weiner: An application of positive definite functions to the problem of MUBs, Proc. AMS, to appear., (2017).

We present a new approach to the problem of mutually unbiased bases (MUBs), based on positive definite functions on the unitary group. The method provides a new proof of the fact that there are at most $d+1$ MUBs in C^d , and it may also lead to a proof of non-existence of complete systems of MUBs in dimension 6 via a conjectured algebraic identity.

31. M. Weiner: Local equivalence of representations of $\text{Diff}(S^1)$ corresponding to different highest weights, *Commun. Math. Phys.* Volume 352, Issue 2, pp 759–772., (2017)

The proof of an almost 30 years old conjecture is finally completed here (till now, though many cases were covered, some were still missing). The statement is about the local equivalence of positive energy representations of the group of diffeomorphisms of the circle corresponding to the same central charge.

32. M. Mosonyi, T. Ogawa: Strong converse exponent for classical-quantum channel coding; *Communications in Mathematical Physics*, 355(1), pp. 373–426, (2017).

We determine the exact strong converse exponent of classical-quantum channel coding, for every rate above the Holevo capacity. Our form of the exponent is an exact analogue of Arimoto's, given as a transform of the sandwiched Rényi capacities with parameters $\alpha > 1$. Our result adds to the growing body of evidence that the sandwiched Rényi are the natural quantities for the purposes of strong converse problems.

33. PE Frenkel, J Pelikán: On the greatest common divisor of the value of two polynomials *American Mathematical Monthly* 124 (5), 446-450, (2017).

It is shown that if two monic polynomials with integer coefficients have square-free resultant, then all positive divisors of the resultant arise as the greatest common divisor of the values of the two polynomials at a suitable integer.

34. P Csikvári, PE Frenkel, J Hladký, T Hubai: Chromatic roots and limits of dense graphs *Discrete Mathematics* 340 (5), 1129-1135, (2017).

It is shown that recent results of Abért and Hubai and of Csikvári and Frenkel about Benjamini-Schramm continuity of the holomorphic moments of the roots of the chromatic polynomial extend to the theory of dense graph sequences.

35. PE Frenkel: Polynomial identities for matrices over the Grassmann algebra, Israel Journal of Mathematics 220 (2), 791-801, (2017).

Minimal Cayley--Hamilton and Capelli identities for matrices over a Grassmann algebra of finite rank are determined. For minimal standard identities, lower and upper bounds on the degree are given.

36. P. E. Frenkel: Convergence of graphs with intermediate density, Trans. Amer. Math. Soc., to appear, (2017).

A new notion of graph convergence, interpolating between the known bounded degree and dense theories, is introduced. It is shown that eigenvalue distributions of the adjacency and the Laplacian matrix behave nicely along convergent graph sequences.

37. S. Carpi, Y. Kawahigashi, R. Longo, M. Weiner: From vertex operator algebras to conformal nets and back, Mem. Amer. Math. Soc., to appear, (2017).

There are several different mathematical descriptions of a conformal quantum field theory. Some researchers considered (unitary) vertex operator algebras, whereas in the operator algebraic setting, one tries to capture essential physical structure by a net of von Neumann algebras. This article contains a detailed investigation of the mathematical connection between the 2 structures (which turns out to be roughly similar to that between Lie algebras and Lie groups).

38. T. Titkos: The singular part as fixed point, Amer. Math. Monthly, to appear, (2017)

The aim of this note is to investigate the classical Lebesgue decomposition of measures from a new point of view. We identify the singular part as a fixed point of a nonnegative finite measure valued map. The proof also shows that Ando's decomposition for bounded positive operators is in full analogy with the Lebesgue decomposition of measures.

39. Tarcsay Zsigmond, Titkos Tamás: On the order structure of representable functionals, Glasgow Mathematical Journal, to appear, (2017).

The main purpose of this paper is to investigate some natural problems regarding the order structure of representable functionals on $*$ -algebras. We describe the extreme points of order intervals, and give a nontrivial sufficient condition to decide whether or not the infimum of two representable functionals exists.

40. V. Morinelli, Y. Tanimoto and M. Weiner: Conformal covariance and the split property, to appear in Commun. Math. Phys. (2017).

In this article it is shown that the split property - an important operator-algebraic property of quantum field theories - is automatic in the conformal chiral setting. This statement was actually conjectured around 10 years ago. The presented proof relies on a delicate analysis of the representation theory of the group of diffeomorphisms of the circle.