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Budapest University of Technology and Economics, Faculty of Mechanical Engineering
Department of Applied Mechanics

Final research report on the NKFI 134303 project entitled

Modern engineering experiments performed on elastic composite structural elements

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1. Models for thin steel and composite bistable shells

Three-roll bending process and curvature measurement of thin plates

We carried out an investigation in the field of bistable shells, and performed experimental research of a manufacturing procedure through the three-roll bending process. The three-roll bending process is a simple procedure, commonly used in the industry, through which a cylindrical surface can be produced from a sheet plate. This process is mainly controlled through experience, and it is described with the finite element method, except for a very few numerical and analytical investigations. We needed a proper way to model this process with a relatively low computational cost, thus a numerical method was found and implemented in MATLAB, which was able to find the curvature function along the arc length of the sheet metal (or beam, depending on the model that is being used). A verification of the model was carried out using the finite element method. The curvature functions of the two models were in exceptional agreement (a maximum of 3.2% in the relative error). While the implementation was done using the Kirchhoff plate theory and a bilinear plasticity model, it could easily be modified to virtually any plate or plasticity model by changing the bending moment-curvature function of the workpiece. It was found during numerical calculations, that the curvature function resembled the time-motion graph of a damped harmonic oscillator, which was not shown in the literature at all. This oscillation of the curvature persisted even if the radii of the rollers were reduced to zero. We published a related conference paper entitled: [*Numerical modelling of the three-roll bending process of a thin plate*](#) in *Műszaki Tudományos Közlemények (Sciendo)* and it was also presented at the International Conference of Young Engineers in 2020.

For a better applicability of beam and plate models, we searched a way to determine the curvature distribution of a slender, bent beam or a thin plate that was bent in only one direction. Thus, a proper procedure was developed which used digital image processing to determine the bent shape, based on a digital image taken of the beam. The optical method was necessary especially for slender beams with low flexural stiffness which could easily change their shape when the measuring tool touched them. However, the best way for such an application was to use parallel projection, it was found that the central projection from a camera did not induce significant distortion if used with a telephotographic lens. The process first distinguished the beam from the rest of the image, then produced a weight map based on the color values of the pixels. By averaging

the weight map one could calculate coordinates of the centerline in distinct points. However, these points were the proper representation of the centerline, because of the discrete character of the image, they had a significant noise, which induced a high error in the further results. This error could be handled by smoothing the given data. This way a proper approximation of the curvature function could be obtained. This curvature function and the centerline were compared to the results of a numerical simulation, they showed a maximum of 5% relative error. The results were presented at the International Conference of Young Engineers in 2021, the conference paper was published in *Műszaki Tudományos Közlemények* (Sciendo) entitled: “[Curvature measurement of thin plates with the help of digital image processing](#)”.

Bistability of steel and composite cylindrical shells

We conducted a research on the analysis of bistable thin cylindrical shells, too. We found that their ability to have two stable forms highly depends on their size and not only on the shaping parameters. For certain size combinations (width and length) they may lose one of their stable configurations. We created a hypothesis that if the integral of the residual internal bending moment along the shell's edge could cause the snap-through of the thin cylindrical section, then the second stable state would cease to exist. For the description of this problem as a first step, we investigated the stability of the thin cylindrical shells without internal stresses. In the literature, these are called tape springs. During the research, we created a simplified, dimensionless analytical model, which described the behavior of these shells for the pure bending scenario. Since the model was dimensionless, it made parameter analysis possible. However, the final solution to the problem required numerical calculations, thus making this method a semi-analytical one. Using this method, we found that the snap-through behavior may not appear for certain cross-section size combinations (dimensionless thickness and arc angle). We also found that there was a single borderline between the dimensionless parameter regions that had and those which do not had the snap-through phenomenon.

For the validation of the model, we conducted finite element simulations. Since the problem was a highly unstable one, it was impossible to simulate the pure bending, thus instead we simulated the three-point bending. Theoretically, the bending moment at which the beam with the circular cross-section snapped through converged to the bending moment of the pure bending scenario when the length went to infinity. Thus, by analyzing the convergence of the moment at the snap-through in the function of length one could compare the simulations with the analytical model. The analysis showed a 15-20% relative error for the value of the critical dimensionless bending moment; however, the critical beam curvature was found to be accurate. The finite element simulations also revealed that a cross-section that had a snap-through in its bending characteristic may not lose stability below a certain length when the loading procedure is displacement controlled. The results of the research were published in the journal *Mathematics and Mechanics of Solids* (Sage) in 2024, the title of the article is: “[Stability analysis of thin cylindrical shells under pure and three-point bending](#)”.

Bistable behavior (snap-through) can be achieved by varying the lay-up of composite shells, i.e., the shell would have two different shapes that are in equilibrium without any constraint. We have investigated the **effect of the stacking sequence on the stability of composite cylindrical shells**. For the investigation, the analytical model of Galletly and Guest was used as a basis. During the investigation, a total of five lay-up structures were used, the two proposed by Galletly and Guest (symmetric and antisymmetric layups, with, and without middle layer), and the

one used by Brunetti et al. The initial research aim was to find lay-ups that would restrict the flattening of the cross-section, however we proved, that no such lay-up may exist, and the shell would always flatten after a critical longitudinal curvature. Moreover, by examining the effect of the orientation angle of the plies we found that for some orientation angles and some geometries the shell does not exhibit stability loss during bending, i.e. snap-through phenomenon may not appear for every geometry and stacking sequence. Unfortunately, for these problems no closed-form analytical criterion was found.

Another research aim was the bistable behavior of these shell types. Based on the original model, we derived a closed-form criterion of bistability for composite cylindrical shells. The criterion involves a combination of the cross-section parameters (radius, central angle and laminate thickness), and states that to achieve bistable behavior, this should be greater than a constant, that is a function of the laminate stiffness parameters. The effect of the orientation angle on the bistability limit was also investigated. We conducted FE simulations and compared them to the analytical results where applicable. The qualitative behavior predicted by the analytical model is in good agreement with the FE simulations. The results were summarized in an article entitled “*The effect of the layup on the stability of composite cylindrical shells*”, that is currently under review (after first revision) in *Periodica Polytechnica, Mechanical Engineering*.

2. Dynamics and fracture of delaminated composite beams

Influence of boundary conditions and time delay on the dynamics of delaminated composite beams

We performed **standard vibration tests on delaminated composite beams** (glass fiber reinforced polyester) including sixteen different configurations regarding the length and thicknesswise position of the delamination. Two different kinds of excitations were applied: the modal hammer test and the shaker (sweep excitation) test. The experimental results were compared to computational results obtained by analytical and finite element models. A novelty of these models was to model the elasticity of the built-in length of the beams. This was achieved by placing linear Winkler foundation (a set of linear springs) along the built-in length. The literature have not yet emphasized the importance of this effect at all. The results showed that because of the built-in end resting on the elastic foundation the agreement between the measurements and the computational models was significantly better than by assuming rigidly fixed beam end. The average difference between the first natural frequencies by experiments and model prediction assuming rigidly fixed end was 15%. By using the modified models we managed to reduce the former difference to 4-5%, which is a significant improvement. We also analyzed the mode shapes and the stress resultants in the beams using the two different approximations and we found that the effect of elastic foundation is moderate in this respect. We also investigated the parametric excitation phenomenon taking place during the free vibration of delaminated beams. We applied two different computational models: the first one was the harmonic balance method with unreduced local finite element model of the delaminated part. The second was based on a modal decomposition technique. Using both models we determined the stability diagrams of the beam configurations. These stability diagrams were to estimate the critical amplitude of the beams the delamination buckling takes place at. The final results showed that the unreduced model predicts better the critical amplitude, however, it is still inaccurate compared to experimental observations.

The results above were summarized in an article entitled „[*On the dynamic stability of delaminated composite beams under free vibration*](#)” and published in *Acta Mechanica*, 2022.

The problem of the **delaminated composite beams subjected to a retarded periodic follower force** was also investigated. The delaminated composite beam was captured by a FE model developed in the paper mentioned in the previous paragraph (*Acta Mechanica*). The periodic follower force involved two time related parameters: the period of the excitation and the time delay. The former means that the nonconservative force at the end of the beam follows the beam end rotation, the latter means that the rotation was captured by the system at a moment prior to the current one. The FE equation of motion was created by considering the geometric and load stiffness matrices and led to a system of integro-differential equations. We got started with the simplest case when the time period and the time delay were equal to each other. In this case we analysed the stability of the system. First, we solved the equation system by using the Chebyshev polynomials. Using the Floquet theory we developed the transition matrix form of the equations for a single period. The stability of the system was determined based on the well-known unit circle criterion requiring the spectral radius of the Floquet transition matrix. Large number of stability diagrams were determined on the different parameter planes including the following: static force - time delay, static force – dynamic force amplitude (conservative force without delay), static force – dynamic force amplitude varying the load direction parameter and the first frequency square – load direction parameter. We elaborated that the static force - time delay diagram is quite sensitive to the number of finite elements (mesh resolution) and it was very time consuming to obtain mesh dependent results. On the other hand the other diagrams were found to be almost insensitive to the mesh. We also investigated the effect of delamination and we observed that by varying its thicknesswise position the stable regions were possible to alter. We published an article in *Archive of Applied Mechanics* (Springer) in 2023 („[*Stability of delaminated composite beams subjected to retarded periodic follower force*](#)”). Also, a presentation on a national conference was performed (MAMEK XIV., Miskolc, Hungary 29.-31. August 2023).

Fracture mechanical experiments on specimens complemented with linear springs

We also carried out the **fracture mechanics and stability investigation of composite specimens with springs**. In the beginning phase we manufactured the required number of specimens out of polyester resin and glass fiber (approximately 30 specimens were fabricated). Then we chose two specimen types, which are relatively easy to measure: the double-cantilever beam (DCB, mode-I fracture) and the 4-point end notched flexure (4ENF, mode-II fracture) specimens. We measured the load–displacement response and tested the specimens upto crack initiation for different crack lengths. In accordance with the literature and an improved analytical model we determined the stability or bifurcation diagrams of these systems assuming constant critical energy release rate (ERR) with the crack length. For the DCB specimens we applied the Timoshenko beam on Winkler-type elastic foundation in order to capture the root displacement and rotation accurately. The system was complemented with springs at the end of the specimen arms and springs covering five different stiffness values (plus the case without the spring or with infinite stiffness) were applied. Depending on the applied spring stiffness the bifurcation diagram consisted of two parts for each crack length. A stable part involving increasing crack length with increasing displacement (subtle bifurcation), and an unstable one involving decreasing crack length with increasing displacement (rigid bifurcation). The measurement results fit quite well the theoretical results (bifurcation curves) in the case of the DCB specimens.

Moreover, we modeled the 4ENF specimen with second-order shear deformation beam theory. We showed that the contribution of the shear deformation is rather related to the compliance and negligible from the standpoint of energy release rate. The 4ENF experiments were performed by using a saddle and several steel pins. For symmetry reasons, two springs were applied parallel to each other (one spring on both sides). Linear springs with four different stiffnesses were applied and for each stiffness value specimens with thirteen different crack length values were applied. This means a total of 52 measurements for the 4ENF test. The measured dataset was evaluated by the analytical model and the well-known compliance calibration (CC) method. The data evaluation revealed that the analytical model provides significantly smaller scatter in the R-curves (critical energy release rate and crack length relationship). In the final stage of the research we calculated the so-called bifurcation curves (critical displacement vs. crack length). The stability of the system was investigated based on the energy release rate expression in terms of displacement and crack length. The results indicated that the so-called subtle bifurcation was expected in the 4ENF test and the critical displacement vs. crack length relation was always linear. In other words the 4ENF test is always stable independently of the stiffness of the attached spring. Moreover, the test cannot be destabilized with additional springs. We summarized the fracture mechanical measurements including the DCB and 4ENF tests in an article. The article was published in the journal *Theoretical and Applied Fracture Mechanics* (Elsevier) in 2023 („[Fracture mechanical stability of DCB and 4ENF tests complemented with linear springs](#)”). The main results of this work were also presented in a conference in Lovran, Croatia (AMSE-2024: The 5th International Congress on Advanced Materials Sciences and Engineering 23-26 July 2024, “*The effect of variable stiffness on the stability of displacement controlled DCB and 4ENF tests*”).

3. Development of plate finite elements for delaminated composite and sandwich plates

Finite element model development for delaminated composite and sandwich plates

We developed a plate finite element model for the analysis of delaminated composite plates. In our models, the von Kármán nonlinearity was taken into account, but the opening of the delamination was not allowed. Three fundamental element types were defined: an intact, a delaminated and a transition element, respectively. The plate elements were based on the first- and third-order shear deformation theories including subparametric and isoparametric interpolation schemes. To carry out a nonlinear FE analysis, specific functions and algorithms were created in a Python programming environment such as an algorithm for the Newton-Raphson iteration and a procedure to calculate the tangent stiffness matrices. Furthermore, the FE formulation of the kinetic energy was also deduced for each element type. Also, we carried out some fracture mechanical computations to verify the developed FE types. This means the determination of the J-integral over the delamination front and comparison to 3D ANSYS solution by the VCCT (virtual crack closure technique) method. The agreement between the two different methods was quite good. The results were published in *Applied Mathematical Modelling* („[Advanced finite element analyses to compute the J-integral for delaminated composite plates](#)”) in 2024. We also took part on the 9th ECCOMAS Thematic Conference on the Mechanical Response of Composites conference (Trapani, Sicily, Italy, 12-14 September 2023) where we presented the basic concept of the finite element model for sandwich plates („[Modelling of delaminated sandwich structures satisfying every dynamical boundary and continuity condition](#)”).

Flutter and divergence analysis of delaminated composite plates

Based on the models that were created for modelling delaminated composite plate structures priory, a novel finite element family was developed by consolidating the first-order shear deformation and piston theories. The models built with these element types are appropriate for modelling and examining **thin airfoils subjected to supersonic or hypersonic flows** from different aspects. The structural arrangement is that the airflow direction is parallel to the middle surface of the delaminated plate. The linear finite element models provided a convenient way to analyse the stability of the structures in terms of boundary conditions, delamination location and delamination size. Varying the Mach number (ratio of airflow speed and speed of sound) and the parameters mentioned above, the stability maps of different cases were generated. In the linear models, stability loss yielded that the displacement amplitudes tended to infinity, however, this phenomenon does not occur in real life because of nonlinear effects. Therefore, invoking the von Karman nonlinearity, the finite element family was improved by considering geometric nonlinearity in the transversal direction. To investigate the nonlinear vibrations of delaminated composite airfoils induced by airflow, the Newmark direct time integration procedure was accelerated and modified which resulted in a fast predictor-corrector method for solving the nonlinear equation of motion. Regarding the stability maps, several simulations were carried out with different parameters related to stable and unstable states. In these cases, the position of the through-width delamination was varied: it was parallel or perpendicular to the airflow direction. The position of delamination over the width of plate was also varied. The results showed that both divergence and flutter instabilities could appear but the type of stability loss depended on the boundary conditions, Mach number and delamination layout. The paper entitled “*Flutter instability of delaminated plates in supersonic and hypersonic flows*” is under submission to *Nonlinear Dynamics* (Springer).

4. Engineering experiments on steel structural elements

Measuring the tension loss in a cable traveling over a pulley

An investigation was carried out to **measure the tension loss in a cable travelling over a pulley**. The initial measurement method consisted of a cable loop going around two pulleys, that was put under different tension levels, and the force breaking the frictional force and moving the cable was determined in function of the preload. At one end of the cable a spring was installed between the cable and load cell, which allowed the pulley to rotate in a predefined direction when being pulled up or down. One gets a theoretically continuous tension loss function in function of the bearing radial load. The measurements had a significant error initially because of the load cell's sensitivity to cross-loads and due to the fact that the tension loss was relatively small compared to the tension at the ends of the cable, measured by the cells. A method was found to eliminate this error, and the tension loss coefficient (defined as the ratio of tension loss and bearing radial load) could be determined with high repeatability. It was proven that the hysteretic energy of a unit cable length calculated from the bending model was equal to the additional tension loss caused by the internal friction of the cables. The results of this research project were presented in an article entitled “[*Measurement of the tension loss in a cable traveling over a pulley, for low-speed applications*](#)”, published in *Experimental Mechanics* (Springer) in 2024.

Bearing friction by a simple pendulum

A novel **method** was developed for **determining the bearing friction** itself, without the need to take the cable's bending properties into consideration. A major factor in this investigation was the lack of the usual bearing friction measuring equipment, and as such, a low-cost and efficient method was proposed. The idea was to use a simple pendulum that was fixed through the bearing under investigation. Furthermore, the movement of the pendulum's endpoint was captured by a smartphone camera, thus the angular displacement could be determined in time by a simple DIC algorithm, from the video footage. The pendulum in this case would have a combined damping with both viscous effects and Coulomb-friction. By finding the local extrema of the angular displacement, one could calculate and plot the decay in each step, in function of the extrema itself. It can be proven that these should give collinear points, and a linear fit will allow the separation of the damping parameters, thus one can determine the bearing's coefficient of friction. The measurements returned the value specified in the bearing catalog. The results were presented at the ASME IDETC/CIE conference (*"Bearing friction measurement with a pendulum"*) that took place in Boston (MA) between the 20th -23rd of August 2023 (Technical Presentation).

Bending device to test elastic beams with large displacement

We created a device which can reproduce the **uniform bending of beams with large displacements**. For this purpose, we created a pulley system that can be attached to a tensile testing machine. This device is able to generate a concentrated bending moment with a good approximation at the end of a beam with no limitation on the displacement of the beam's end. The other end of the beam was always fixed. Our solution for this task was to clamp two beams together and bend them in opposite directions, thus generating a fixture which does not let the beam's end to rotate. However, the fixture was able to move on the symmetry line between the two beams. As the design needs the test specimens to be horizontal in the unloaded state, and specimens with significant weight can also be tested, the fixed end needs to be supported horizontally, which was done with the help of a linear guide. Clamping the two beam ends together and bending them in opposite directions helped also in reducing the bending moment on the linear guide, which helped in reducing friction in the system. To guarantee that the two forces on the pulley generating the bending moment are equal, the two strings coming out of this pulley were attached, thus forming a loop around four other pulleys. While loading the beams, one of these pulleys is being pulled by the tensile testing machine. The pulley generating the bending moment and the pulley attached to the tensile testing machine was not fixed to the device, unlike the other three which were at fixed distances from each other. This way the pulley pulled by the machine was forcing the loop to enlarge its circumference. If there is little or no strain in the string, it will force the moment-generating pulley to rotate. Since the string was only fixed to the beam end and it could run smoothly around the other pulleys, the beam end's position was only defined by its deflection. This way it truly can behave as a beam loaded by a concentrated moment which is fixed at the other end.

For a deep understanding of the machine, several measurements were conducted with 3x20 mm hot-rolled steel bars. The design features of the machine are the following. Maximal beam length: 500 mm. Maximal displacement: 250 mm. Maximal rotation of the beam's end: 135°.

Maximal beam thickness: 5 mm. Maximum torque: 20000 Nmm. At each load value, a photograph was taken of the specimens, onto the side of which equidistant white dots were painted previously. A MATLAB algorithm was written to find the location of the marks and estimate the beam shape. The point locations were used for circle fitting, which yielded the curvature of the beam. The bending moment was calculated initially from the measured force on the tensile testing machine. This method led to a hysteretic bending moment- curvature characteristic, due to the imperfections caused by the friction in the system. We also measured the approximate tension loss on the cable when it travels over a pulley to have a better estimate of the bending moment and the parasitic concentrated force on the beam end. We also found that, while the marker locations are remarkably close to a common circle, it is still significant, that the shape of the beam is not circular. We found that in the domain of these tests, if one fits a circle to a point set which is a discrete representation of a function with a linear curvature in the horizontal distance from the beam end, then the curvature of the fitted circle will coincide with the curvature close to the middle of the horizontal domain. To summarize, the circle fit would yield the curvature in the middle of the beam, while the bending moment calculation represents the moment at the beam end. It was found that if one calculates the bending moment in the middle of the beam and plots it against the curvature value from the circle fit, then the hysteresis can be eliminated. In the plastic region we were also able to create a plastic hinge on a relatively large portion of the beam (near the loading pulley end) and measure the plastic moment accurately.

We determined the bending stiffness values from the measured curvature- bending moment characteristics with the reduced hysteresis, and it showed good agreement with the ones obtained from regular three-point bending measurements. We wrote an article entitled “*A pure bending device to test elastic beams in the nonlinear region*” and it is under submission to *Experimental Mechanics* (Springer). The results by using the bending device were presented at the MAMEK XIV. (Hungarian Mechanics Conference) in Miskolc, Hungary 29.-31. August 2023.

Three-point bending with frictionless contact and large deformation

As a part of a newly designed three- and four-point bending device we invented **a support, that is able to simulate an edge support** (i.e. the contact line between the specimen and support remains static), without, or at least with negligible friction. The usual supports used for bending tests are cylindrical (either static, or rolling), on which the contact line may change its location, and thus, the span length, especially in the case of large deflections. Smaller radii may be applied, but those could dent the specimen, and impose extra tangential force at the contact points onto the beam that is being tested. Our solution for keeping the contact static and eliminating friction simultaneously was the following: the specimen is placed on a roller, that has two ball bearings installed at both of its ends, functioning as wheels. These bearings roll on a cylindrical surface, the axis of which coincides with the contact line. The dimensions should be chosen so that the rolling axle's surface should touch the axis of the path. This way, the specimen should be able to move freely on the rolling axle, while its contact with the rolling axle should not change location, for any rotation of the beam ends. The support was manufactured as part of the bending device and was put to test. During measurements photographs were taken of the beam under bending, and the bearings' visible sides were painted with a random pattern so that their rotation could also be measured using DIC (digital image correlation) algorithms. The beam shape was also measured using a DIC algorithm. The measurement results are in perfect agreement with the simulations

conducted on the nonlinear three-point bending with frictionless edge support, which prove that the design worked as expected. The results were presented at the 40th DANUBIA-ADRIA SYMPOSIUM on Advances in Experimental Mechanics held in Gdansk, Poland. We plan to submit an article to an international scientific journal, too (after conducting further measurements).

5. Differences compared to the original research plan

Unfortunately, we had many difficulties to realize the original research plan. Nevertheless, we would like to emphasize that during the 4 years of the research we always found an alternative task if an original one was abandoned for any reasons. In the following we give the most crucial points we faced during the research.

- Although we built the small-size wind tunnel and we performed many experiments on aluminium plates subjected to airflow, we recognized that the performance of the axial fan and the size of the tunnel was insufficient to reach the condition of flutter instability. On the other hand, the vibration response of the system was quite sensitive to the imperfections of the experimental scenario (airflow speed fluctuations, boundary conditions, etc.). Thus this part of the research work was abandoned.
- The experimental proof on the appearance of microcracks in sandwich specimens was also abandoned due to the high complexity of the task. We would like to mention similar reasons for the case of parametric excitation and piezoelectric experiments under different boundary conditions and the tests of piezoelectric beams.

On the other hand, we believe that instead of the above mentioned experiments we provided others (refer to the report) that fit better to the qualification of the participants and we also provided some important results related to numerical modelling of composites. We are convinced that these results will form a good basis to write and submit the PhD theses by Péter Máté and Bence Hauck within a short-term period.

Below, we provide the original workplan and the modified one for clarity.

Period	Original workplan, tasks	Modified workplan, tasks
1. 2020.09.01- 2021.08.31.	Flutter and divergence of composite beams, wind tunnel experiments	Experimental frequency analysis of delaminated composite beams
	Analysis of bistable steel and composite shells	Curvature measurement on thin cylindrical shells
2. 2021.09.01- 2022.08.31.	Large displacement experiments on elastic beams	Fracture mechanical experiments on specimens with linear springs
	Fracture mechanical experiments on specimens with linear springs	Curvature measurement on thin cylindrical shells
3. 2022.09.01- 2023.08.31.	Experiments on sandwich 4PBP and ECT specimens, microcracks	Development of a cable-pulley system to measure the tension loss of cable wires
		Development of a pendulum based

	Experiments on parametrically excited composite beams with various boundary conditions	measurement method for the coefficient of friction in ball bearings
		Modelling delaminated beams subjected to retarded periodic follower force
		Finite element model development for delaminated composite and sandwich plates
4. 2023.09.01- 2024.08.31.	Modelling of beams and plates with piezoelectric layers, simple experiments	Numerical (FE) analysis of flutter and divergence in delaminated composite plates
		Analysis of steel and composite bistable shells,
		Large displacement experiments on elastic beams
		Three-point bending with frictionless contact and large deformation

Finally, the support of NKFIH is greatly acknowledged during the past four years. Thank You.

Budapest, 30th September, 2024.

Dr. András Szekrényes