

Project Summary Report

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In the course of this project we have used a very versatile effective model of QCD, the vector and axial vector extended linear sigma model (eLSM), to tackle various theoretical problems related to the description of the strong interaction in the medium. First, we studied the baryon fluctuations in a so-called hybrid approximation, which means that mesons are treated at the tree-level, while fermions are treated at the one-loop level. Accordingly, susceptibilities of different orders were calculated and compared with the corresponding lattice result [1].

We have studied the low temperature, high density part of the phase diagram of strongly interacting matter. The matter of neutron stars, which are extremely dense stable astrophysical objects, occupies this regime of the phase diagram. Their central density is estimated to be about 2-5 times the nuclear saturation density, while their temperature drops below 100 eV in a few years due to neutrino cooling. In our investigations, we aimed to apply our model to high-density matter in neutron stars. In the last decade, the observation of neutron stars has regained momentum. In addition to the observation of heavy pulsars with masses of about 2 solar masses, the NICER experiment, which aims to simultaneously measure the masses and radii of neutron stars from their X-ray pulse profiles, was launched. Also in 2017, the LIGO and Virgo detectors observed the gravitational wave produced by a binary neutron star merger. With rapid localization, astronomers were able to observe the event in a wide range of the electromagnetic spectrum. This provided important information about the properties of the merger. While in the past the lack of precise observations allowed for a wide range of nuclear models, today all models should be tested for compatibility with these observations.

Since the outer core of neutron stars consists of nuclear matter, i.e. strongly interacting matter, and as the density increases towards the interior the strongly interacting matter might deconfines (hadron-quark transition), one needs to describe the nuclear matter in their model together with an appropriate approximation for the hadron-quark transition. We explored several possibilities for realizing such a phase transition, including statistical confinement, the energy minimization method, and pressure interpolation. After carefully studying these possibilities, we finally decided to use concatenation methods, which apply a smooth transition between the two regions. This allowed us to parameterize the location and strength of the transition. The first results for a hybrid star, i.e. a neutron star with a quark matter core, within the eLSM were published in [3, 5].

We have developed numerical codes to calculate several important properties of neutron stars, given a particular equation of state. One of these properties is the quadrupole tidal Love number, which describes how much a neutron star is deformed when exposed to an inhomogeneous external gravitational field. This property is most relevant in the context of gravitational wave observations, where a combination of the tidal deformabilities of two merging neutron stars can be measured directly. Regarding this quantity, we found that when considering equations of state that include a first-order phase transition, the literature uses an incorrect formula for the term to be added at the phase transition point. We checked the difference between using the incorrect and the correct formula and found it to be only about 5%, which is negligible considering the current accuracy of the measurements, but may become significant when they are improved. We published the correct formula in [2] as a reflection on the paper in which the erroneous formula first appeared. Our code is also able to calculate the

mass and radius corrections for a rotating neutron star with a given central energy density and a rotational frequency low compared to its Kepler frequency (the limit frequency at which the neutron star is still stable). These corrections can become important as the precision of the measurements improves, since radius measurements are aimed at millisecond pulsars, for which the deformation due to centrifugal forces can become non-negligible.

Next, we have studied the renormalized fermionic one-loop contribution to the vector and axial vector meson curvature masses with different methods. It was found that such a regularization has to be chosen that takes into account the vacuum properties of the vector self-energy, resulting from the divergenceless nature of the vector current for certain flavor indices on a specific scalar background. Due to the tensor structure of the fermionic self-energy for the (axial) vector mesons, a mode decomposition was performed at both zero and finite temperatures. This procedure revealed further details of the mixing between the (pseudo)scalar and the (axial) vector fields. We studied the thermal behavior of the obtained in-medium curvature masses of the different modes, which showed an increase deep in the chirally symmetric phase only for the 3-longitudinal mode. The improvement of the model helped to understand several properties of the quark-meson model with (axial) vectors and also outlined the possible application of the so-called Gaussian approximation within the eLSM. Our results have been published in [4].

We have also studied the N_c dependence and the large N_c limit of the eLSM. The phase diagram was studied for finite chemical potential and also for finite temperature. For the latter, we had to implement a more general N_c -dependent Polyakov loop potential. In order to reduce the number of degrees of freedom, which scales linearly with N_c for $T > 0$, the so-called uniform eigenvalue ansatz was applied. It was found that the well-known $N_c=3$ critical endpoint (CEP) is already absent at $N_c=4$. The chiral and deconfinement phase transitions are separated for high chemical potentials, and a confined but chirally symmetric phase appears, which can be identified with the quarkyonic phase. As the number of colors increases, a second CEP appears next to the temperature axis at $N_c=53$ and then moves to higher chemical potentials. For large N_c , the critical temperature and the pseudocritical chemical potential saturate, while the new CEP draws a first-order line at the saturating critical temperature. Above this phase boundary one finds the usual deconfined and chirally symmetric phase, while below it there remains the confined and broken hadronic phase at low chemical potentials, separated from the symmetric but confined quarkyonic phase by a crossover chiral transition. Our results have been published in [6] with the Editors' Suggestion.

We have improved our model to be able to apply it in the zero temperature and finite density regime. Instead of our previous simplified approximation, we have separately included the vacuum expectation values of the three vector mesons that are expected to condense. These vector meson condensates affect the quark content of high-density matter by modifying the chemical potentials of the quarks to varying degrees. With the inclusion of these vector meson condensates, however, a new problem arose. Using a naive parameterization of the model, we found that these condensates modify the field equations in such a way that the chiral symmetry is broken at high densities after being partially restored at intermediate densities. To solve this problem, we investigated the asymptotic behavior of the field equations as well as the condensates, and required that the order parameters of the chiral symmetry, i.e. the scalar condensates, tend to zero. We found that these requirements can be satisfied with an additional parameter constraint [7]. With this additional constraint, we were left with the g_V vector

coupling as the only free parameter that is not constrained by measurements at the level of approximation we used. However, since the sigma meson is an experimentally broad resonance and its optimized model value was far from its experimental value, we decided to leave this parameter unbound as well. We have studied the order of the chiral phase transition at finite density and zero temperature as a function of these parameters, the g_V vector coupling, and the sigma meson mass. We found that a first-order transition is only possible with the sigma meson mass chosen as the lowest value in our set, 290 MeV, which is also the optimal value taken from our original parameterization. We also found that increasing g_V turns the first-order transition into a crossover at about 3.1.

We have calculated how changing g_V and the mass of the sigma meson affects the masses, radii, and tidal deformabilities of hybrid stars. We also studied the effect of using different hadronic equations of state and different methods for concatenating the hadronic and quark parts. We found that the maximum neutron star mass is only weakly dependent on these, and can be used to constrain the parameters of the eLSM, i.e. the vector coupling and the sigma meson mass. We have shown that for a sigma meson mass of 290 MeV, the value of the vector coupling is constrained between 2.5 and 4.3. Taking into account our previous result on the order of the phase transition, we have also shown that the chiral transition at zero temperature can only be a weak first order or a crossover transition in the framework of our model. We have also published these results in [7].

In September 2021, during a Short Term Scientific Mission (STSM) to Wroclaw, a new research direction emerged: the system size dependence of thermodynamic quantities and the phase diagram, in collaboration with the Elementary Particle Theory Department of the University of Wroclaw. The finite spatial extent is an important feature in heavy ion collisions, so there are attempts to include its effects in field theoretical models that are in the thermodynamic limit, where the volume is infinite. For this purpose, it is common to include only the modification of the momentum space, i.e. a discretization with modes determined by the boundary conditions, or a low-momentum cutoff. However, previous calculations within different models using different approximations and varying momentum constraints have led to inconclusive and sometimes even contradictory results. Therefore, we have applied both the momentum cutoff and the discretization with the commonly used (periodic and antiperiodic) boundary conditions within eLSM to understand the difference between the approximations and the resulting modifications. We found that the different constraints and boundary conditions can give substantially different results, e.g. for the trend or even the existence of the critical endpoint for small system sizes. Moreover, we have shown that the effect also strongly depends on the treatment of the vacuum term. Our results can explain certain differences in the literature, while also highlighting the need for a more sophisticated approach to study the finite volume effects. We have also studied the size dependence of the baryon fluctuations in the vicinity of the CEP, which is one of the candidate observables to identify criticality in the heavy ion experiments. We found only moderate finite-size effects for these quantities, with the changes most likely related to the change in the location of the CEP. Our results can be found in [8,9].

With all the previous considerations for the equation of state taken into account, we constructed a large number (around 20 000) of hybrid equations of state, using two different hadronic models and changing the four undetermined parameters (two parameters from the eLSM, and the two parameters of the concatenation). These equations of state were then utilized to calculate various observable properties of neutron stars, which were then compared to

astrophysical observations. We developed a Bayesian framework to investigate the posterior probability of our model parameters given different combinations of astrophysical observations. Using our statistical ensemble, we were able to determine the parameter combinations with the highest posterior probabilities for different subsets of our parameter space. We found that a peak in the speed of sound above the value at conformal densities is highly favored. The position of this peak was determined to be about 3-5 times the saturation density. This suggests the existence of a decrease in the speed of sound, possibly indicating a transition to another phase. We also showed that the matter inside the heaviest neutron stars shows signs of conformality, based on conformality measures recently introduced in other studies. While these may indicate the existence of conformal matter in heavy neutron stars, we pointed out that some purely hadronic equations of state may also behave similarly, given these conformality measures. In addition, we showed that pure quark matter can only exist in our model for neutron star sequences with a maximum mass below 2.3 solar masses. We have published these results in [10, 11].

The effect of isospin breaking in eLSM in vacuum has been studied. In accordance with the three non-zero external fields, namely the nonstrange, strange, and isospin fields, there are three non-zero condensates. The particle mass mixing pattern is more intricate than in the isospin symmetric case, which can be resolved through various field transformations. The calculated physical meson masses, decay widths, and decay constants are compared with their corresponding PDG values through a chi-square fit. A very good fit is found if we exclude the very small (~ 130 keV) $\omega \rightarrow \pi\pi$ decay. We also investigated the violation of Dashen's theorem. The results are presented in [12].

We have also started to study the so-called Gaussian approximation of the eLSM. In this approximation we aim to include mesonic fluctuations in a self-consistent way with a ring resummation, which is expected to modify the thermodynamics and the emerging phase structure. We have found so far that the approximation, when applicable, slightly affects the thermodynamics by lowering the transition temperature and adding a mesonic partial pressure, which further improves the consistency of the temperature dependence of the full pressure with the lattice results. This research is still under development and we plan to publish our results in the near future. In another ongoing study, we are collaborating with Prof. Bence Kocsis at the University of Oxford to investigate the impact of tidal interactions on the evolution of eccentric neutron star binaries. Our findings include the derivation of an analytic formula for the phase shift caused by dynamical tides, which are excited during each pericenter passage of the binary neutron star system. We have demonstrated that in many cases the magnitude of these phase shifts can be high enough to be observable using the current network of gravitational-wave detectors. We proposed that by observing this phase shift, we could constrain the fundamental mode frequencies of neutron star tidal oscillations, which would provide an independent constraint on the equation of state. A manuscript is currently under preparation and is expected to be submitted in the near future.

Our findings were presented at a number of academic conferences, with 32 oral presentations and 11 poster presentations. Finally, both Gy    Kov  cs and J  nos Tak  tsy successfully defended their Ph.D. theses with the highest possible grade, *summa cum laude*, in 2024.

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