Final Research Report

Title of proposal: *New findings on the growth mechanism of thin films and tribological characteristics*

Identifier: K_18-129257 **Duration of project:** October 1, 2018-March 30, 2024

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Place of implementation: University of Miskolc (Miskolci Egyetem ME)

1. PROJECT DESCRIPTION AND STATUS

1.1. OVERVIEW

We summarize the tasks according to the work plan of the original proposal in **Table 1**.

Table 1. An overview of project completion until the end of the project (March 30, 2024).

	Short description	**Methods**	**Related publications***
1. Surface morphology, roughness – tribological properties	Growth models (Kardar-Parisi-Zhang, Kuramoto-Sivashinsky, Ehrlich-Schwoebel-Villain) and noise terms.	analytical (self-similarity, travelling wave), numerical (MATLAB)	17,43,21,*16*, *12*,13,*14*,*15*, 36,*71*
	Friction properties of sputtered TiC/amorphous C	experimental	
	Surface morphology of graphene layer on SiO_2 nanoparticles	analytical	Guedda
	Tribological properties of Ni-P and Ni-B coating	experimental	Bányai
	Coatings on artificial knee joint components	numerical	82
2. Viscosity	Viscosity of glycerol-MWCNT nanofluid	experimental	4
	Viscosity model for silicon oil	theoretical, experimental	33,*58*
	Viscosity equations for mono and hybrid nanofluids	theoretical	45,57, Klazly
3. Solution techniques	Analytical	exact, HDM, HAM	3,6,9,66
	Numerical	Fourier spectral collocation, leapfrog-hopscotch etc.	30,35,53, Kovacs
	Simulations	CFD, FEM	27,29,*34*
4. Pipe /channel flows	Flow in a backward-facing step channel	numerical	*26*,*46*,47,48,63
	Flows in a pipe with heat flux.	numerical	52,*69*,*70*, Khalili
5.Nanofluid flow above flat, curved and rough surfaces	Newtonian nanofluid flows above a porous stretching surface in a magnetic field	Exact solution, similarity method, HAM, CFD	22,*25*,27, 28,59,65,67, 68,72,75,76, 80,81
	Non-Newtonian nanofluid flows above a porous stretching surface in a magnetic field	Exact solution, similarity method, HAM, CFD	20,39,40,41,42, 56,61,66,73, 74,76,Barna,Saeed
	Turbulent flow above blade riblets	numerical, experimental	77, Ali, Hla
6. Ferrofluids	Ferrofluid flows in a magnetic field	analytical, numerical	2,3,5,*7*,18,19

***** Articles published or accepted, submitted or under revision

1.2. PUBLICATIONS

Since the start of the project, **82 papers, 53 papers** in Scopus/WoS-listed journals (total IF: **144.754**) and 5 papers in Hungarian journals have been published or accepted for publication. Six papers are currently under review and three other papers are submitted in Scopus/WoS-listed journals. We gave 8 talks (two of them invited) at international scientific conferences and three talks (one invited) at Hungarian conferences. Two PhD dissertations were defended and another one is under review related to this research and two student papers partly funded by the grant reached first place at the institutional student's research competition in 2019 and 2023 (TDK).

1.2.1. Papers in Scopus-listed journals

Over the whole project period, 53 Scopus/WoS-listed articles were published, with details:

8 D1 [6,20,28,43,66,74,75,80], 10 Q1[30,81,40,45,47,48,56,65,72,79],

22 Q2 [3,4,10,11,13,17,18,19,21,22,24,29,33,35,44,51,53,54,59,67,73,77],

8 Q3 [5,31,32,49,50,61,68,76], 5 Q4 [2,25,26,27,36]

1.2.2. Manuscripts ready, submitted or under revision in ISI-listed journals

9 papers prepared, but not yet published during the project period are available on the shared drive: https://drive.google.com/drive/folders/1VuwMij1K6OsKLmerhRSquhqxVgJvdekW?usp=sharing

- [Klazly] M. Klazly, G. Bognár: A novel empirical effective viscosity equation for hybrid nanofluids, (under review: J. Molecular Liquids)
- [Khalili] M. Khalili, K. Hriczó, G. Bognár: Impact of CuO-MWCNT Nanoparticles on the Heat Transfer of Oil Nanofluid in a Pipe (under review: Results in Engineering)
- [Ali] Z. Ali, G. Bognár: CFD Study of the effect of surface roughness on the ship resistance (under review: Journal of Nonlinear, Complex and Data Science)
- [Barna] I. Barna, L. Mátyás, K. Hriczó, G. Bognár: Analytical investigation of time-dependent two-dimensional non-Newtonian boundary layer equations, (under review: Fluid Dynamic Research)
- [Guedda] M. Guedda, K. Hriczó, G. Bognár: A mathematical model for wrinkling instability of graphene (submitted)
- [Bányai] K. Bányai, M. Czagány, S. Kovács, G. Bognár: Investigation of tribological characteristics for Ni-P and Ni-B coatings (manuscript)
- [Hla] Z. Ali, H. Daoud, G. Bognár: Experimental study on the influence of riblets on drag resistance (manuscript)
- [Kovács] E. Kovács, I. F. Barna, G. Bognár, L. Mátyás and K. Hriczó: Analytical and numerical study of diffusion propelled surface growth phenomena (under review: Parc. Diff. Equ. in Appl. Scie.)
- [Saeed] E.A. Algehyne, F.M. Alamrani, A. Saeed, G. Bognár: A Numerical Analysis of Heat and Mass Convection in a Ternary Hybrid Nanofluid Flow on a Stretching Curved Surface (under review: Journal of Computational Design and Engineering)

1.2.3. Papers in Hungarian journals

In Hungarian journals three papers have been published ([8,40,41]), among them in Hungarian language is [41].

1.2.4. Presentations at scientific conferences

In the reporting period, we have tried to promote our achievements in as many areas as possible in Hungary and abroad. Number of conference papers is 25, these are indicated in italics in the report.

1.2.5. PhD-dissertations

- Mohamed Klazly: The impact of nanoparticles on the fluid flow properties, supervisor G. Bognár (with result summa cum laude on 13.10.2022)
- Okhunjon Sayfidinov: Analysis and evaluation of coating characteristics, supervisor G. Bognár (with result summa cum laude 11.07.2023)
- Mohsen Khalili Najafabadi: Characterization of fluid flow properties of nanofluids in a pipe, supervisors G. Bognár and K. Hriczó (the thesis is at the opponents)

2. SUMMARY OF OUR RESULTS

2.1. The investigation of surface structures, surface roughness, drag force and tribological properties above rough surfaces

The evolution of surface morphology during MBE growth results from an interplay between the deposition of atoms on the surface and the relaxation of the surface profile through surface diffusion. We examined a generalized singular equation to discuss the coarsening of growing interfaces, in the presence of an *Ehrlich-Schwoebel-Villain* (ESV) barrier that induces a pyramidal or mound-type structure without slope selection. We obtained results on the coarsening process by inspecting the behaviour of the branch of the steady state periodic solutions in [17]. We studied the one-dimensional dynamics of a molecular beam epitaxy (MBE) growth in which the mounds increase in both height and lateral size in a wide class of surface growth phenomena. We generalized the result by Hunt et al. obtained for $n = 1$ to $n > 0$ [Hunt AW et al. (1994) Instabilities in MBE growth, Europhys. Lett. 27 611]. For the investigation of the surface morphology of rough surfaces, we made a review on the reduction method by using a model of crystal growth. The principal model used in our investigation describes the evolution of the surface morphology during MBE growth in the presence of an ESV barrier that induces a pyramidal or mound-type structure without slope selection [43].

For the investigation of the *Kardar-Parisi-Zhang* (KPZ) model, we applied two methods. With an appropriate change of variables applying the self-similar Ansatz, we obtained analytic solutions for the KPZ equation for one spatial dimension with numerous noise terms. We investigated four power-law-type noise terms called the brown, pink, white and blue noise, respectively. Each describes completely different dynamics. Additionally, we investigated the properties of Gaussian and Lorentzian noises. Providing completely dissimilar surfaces with growth dynamics [13]. In [12], the same physical phenomena were investigated with the travelling wave Ansatz.

The noise term is involved in the model equation and analytic solutions to the KPZ interface growing equation with various noise terms were investigated in one spatial dimension analytically [13] and with the travelling-wave method in paper [12] ([*14,15*]). Numerical simulations of stochastic equations were performed by preparing code in MATLAB [36,*9*]. Our different choices for the initial condition are applied to the model, it may result from a machining defect in the starting surface similar to the wavy initial surface. The change of amplitudes in the initial condition and/or strength of noise in the noise term has been analyzed.

Results on the coarsening process in the evolution of the surface morphology during MBE have been published in [*16*,21]. The model is based on the nonlinear differential equations by *Kuramoto–Sivashinsky* (KS) and on conserved KS. Our investigation provides the film height as a function of space and time. We have analyzed evolution equations in the context of amorphous thin film growth. It is found that the dynamics depend on the initial wavelength and amplitude.

The aim of one of our works was to find the relationship between the structural behaviour of sputtered TiC/amorphous C (TiC/a: C) *thin films and friction properties.* The 400 nm thick nanocomposites were deposited by DC magnetron sputtering on Si substrates with different Ti power at room temperature. The 6 samples were prepared by the Institute of Technical Physics and Materials Science Centre for Energy Research ELKH. We investigated the friction and wear of coated wafers by the Falex Tribology NV – Belgium. The friction and wear of different TiC coatings were measured with a high-precision Basalt N2 meso-load tribometer. A hard Al_2O_3 tip was used as a counter material and the tests were performed under reciprocating sliding conditions. In the comparison of the average coefficient of friction per cycle, all values per cycle were considered. Surface imaging was made by 3D confocal microscopes. A relative ranking of the coatings and Si substrate reference was achieved in terms of friction and wear depth. Under the selected conditions, the dominant wear mechanisms were presumably abrasion and micro-cutting.

The surface morphology of the graphene layer was also investigated. Our interest is the effect of the spatial structure of the substrate on the morphology of the graphene. We have prepared an experimental investigation of the wrinkling of a graphene membrane supported on SiO_2 substrates with randomly placed topographic perturbations produced by SiO_2 nanoparticles. The analytical and numerical investigations are made using the minima of the stretching, bending and adhesion energy. The wrinkle along the ridge between the two nanoparticles is followed by a smooth one and improves the critical wrinkle length. The paper [Guedda] is submitted, but the experiments are not finished as the AFM has failed and is under repair.

To examine *the effect of the morphology of the coatings on tribological properties*, we prepared types of electroless Ni-P and Ni-B coatings on C40 steel samples at ME. HV hardness was measured on the samples, friction factor and wear volume were measured and calculated, and coating thickness and surface morphology were determined based on SEM images (see paper [Bányai] and a TDK work).

We focused on the coating materials used for artificial knee joint components. The commonly used materials as coatings (titanium-nitride, tantalum, diamond-like carbon, nanocrystalline diamond, graphite-like carbon, hydroxyapatite, and polyether ether ketone) were examined in [82] along with the commonly used techniques for coating the different components of artificial knee joints (*Patent application* No.12907059 was submitted on 12 February 2024 to the European Patent Office).

2.2. Viscosity equations for mono- and hybrid nanofluids

The viscosity of glycerol and glycerol-based nano-fluids containing 0.1–1.0 wt.% MWCNTs, at various temperatures (23–55°C) and rotational speeds (20– 400 rpm) were investigated experimentally, to investigate the rheological behaviour of these fluids [4]. The multi-walled, bamboo-shaped carbon nanotubes were used as additives. Viscosity measurements were performed using a FANN rotational rheometer "50 RCOxp" at ME. The amount of MWCNTs shows a noticeable influence on viscosity, regardless of the applied rotational speed. All tested fluids show exponentially increasing viscosity trends with the increase of MWCNTs amount. The shear stress is linearly proportional to the applied shear rate for glycerol, confirming it is a Newtonian fluid. Nano-fluids containing MWCNTs show non-Newtonian, shear thinning behaviour, the fluids are pseudoplastic [4], (TDK).

The tribological properties and *wear prediction* were carried out with lubricant gear oil (ISO VG 220) on aluminium nanocomposites. We found that the prediction of wear rate, by the regression model and Taguchi analysis, was close to the experimental values [11]. We want to apply this test method for glycerol solutions in the future.

In [45,*57*], we proposed a new empirical equation for the *effective viscosity of mono nanofluids* containing one type of nanoparticle. The equation is based on both theoretical principles and experimental data. It aims to provide a more accurate prediction of nanofluid viscosity, which is crucial for various engineering applications involving nanofluids. The study contributes to the process of improving the understanding of nanofluids in heat and mass transfer processes by providing a reliable tool for estimating their viscosity. Note that the equation is independent of the base fluid. A second paper [Klazly] introduces a new viscosity equation specifically tailored for *hybrid nanofluids*. This equation is derived from experimental data and incorporates Brownian motion as an additional component influencing the mixture viscosity. By integrating individual Brownian motion with correction factors into the proposed models, the equation accurately predicts the effective viscosity of hybrid nanofluids. This research addresses the need for accurate viscosity predictions in the design and optimization of nanofluid-based systems.

For numerical studies of silicone fluids used in viscous torsional vibration dampers, it is essential to have precise rheological knowledge and description of the silicone oil viscosity. In [33], viscosity data from a previous rheological measurement on AK1,000,000 silicone oil were processed. By nonlinear regression, *a temperature-dependent Carreau-Yasuda non-Newtonian viscosity* model was generated. By implementing this parameter set into CFD software, a temperature- and shear rate-dependent viscosity model of silicone fluid was tested using transient flow and thermal simulations. The thixotropy of silicone oil has been measured at Bay Zoltan Nonprofit Ltd. Miskolc. The Weissenberg effect was detected in [*58*] and was applied in ANSYS simulations.

2.3. Solution techniques (analytical, numerical, CFD, HDM, HAM)

We developed numerical codes in Matlab or Maple to find numerical solutions Simulations for publications [3,8,*9*] were performed in MAPLE. Numerical algorithms for the Higher Derivative Method (HDM) are worked out in [3,5]. Numerical simulations were performed in MATLAB [6,10] with the Homotopy Analytic Method (HAM) [66,76,80,81] and for CFD simulations [27,*34*].

We analyzed incompressible and compressible time-dependent boundary layer flow equations with a heat conduction mechanism. The velocity fields can be expressed with the error functions (in some special cases with *Gaussian functions*) and the temperature with the *Kummer functions* [*38*,*39*,40,41]. In [32], we introduced the *self-similar Ansatz* for fluid models describing physically relevant disperse or dissipate solutions containing the error function.

Regarding diffusion, for long times, different types of decay rates are possible for different non-equilibrium systems [*24*,*38*,44,51]. We wanted to understand the different long-time decays for different diffusive systems with comparisons of 14 different numerical schemes [53].

For the numerical examination of a heated ferrofluid flow in a magnetic field the governing equations are simplified under usual boundary layer assumptions and HDM codes were worked out [*1*,2,7].

Numerical codes have been developed to study the self-organisation of surface structures and to understand fundamental physical principles and mechanisms [30]. Both 1D and 2D generalized KS equations were solved with periodic boundary conditions using *Fourier spectral collocation in space and the fourth-order Runge–Kutta exponential time differencing scheme* for time discretization. Computational techniques were developed in the paper [35]. Novel numerical algorithms are constructed to solve the heat or diffusion equation. We started with *different leapfrog-hopscotch* (LH) *algorithm combinations*. We narrowed this selection down to five subsequent tests and demonstrated the performance of these top five methods in the case of large systems with random parameters and discontinuous initial conditions by comparing them with other methods.

The KPZ equation was examined using the LH method, the most *standard forward time-centred space (FTCS) scheme and the Heun method*. A new approach was proposed for the KPZ equation with noise terms to overcome the difficulties of the conventional schemes [53, 54, *55*]. The height distribution function of the 1D KPZ model with different initial condition amplitudes and different noise terms was numerically analyzed in [37] developing a code *in MATLAB.*

In [22,31], *both similarity analysis and CFD simulation*s were performed for the investigation of the impact of the volume fraction of nanoparticles on a nanofluid flow over sheet surfaces. The difference between the two types of models was discussed. In the classical similarity method, we took the usual Blasius neglects in the members of the

Navier–Stokes equation, the governing equations were transformed by the similarity method. For the CFD simulation solutions, the complete Navier–Stokes equation is used. The impact of several parameters controlling the velocity and temperature distributions is calculated by both methods, and the two types of solutions were compared for water-based nanofluids with three types of nanoparticles (Al_2O_3, Fe_3O_4, and TiO_2).

The coupled solid-fluid problem has been investigated by *Finite Element Method* (FEM) using a thermal model for lubricants and surfaces [29].

CFD simulations with the RANS equation and k-ωSST models were used for the investigation of turbulent flow above rough surfaces in [77] and [Ali].

2.4. ***Nanofluid flow in a channel/pipe***

Numerical investigations were performed for different nanofluid flows. We have investigated the application of Al_2O_3 and TiO_2-water nanofluids in backwards-facing step (BFS) geometry with heat transfer for different volume fractions and Reynolds numbers, the heat transfer effects were studied using the Performance Efficiency Index (PEI) [*46*,48].

CFD simulation results for *single-phase and two-phase models* with constant and temperature-dependent thermophysical properties for the heat flux of the BFS problem were compared. Our results showed that the temperature-dependent models resulted in higher heat transfer coefficients and Nusselt numbers and lower temperatures in Al_2O_3-water nanofluids [*26*,*34*,47]. In [48], a temperature-dependent separation flow model was investigated with simulations for the nanofluid separation flow in the recirculation and reattachment laminar flow in a micro-sized backward-facing step channel for water-based nanofluids containing Al_2O_3 and TiO_2 using two-phase model [*60,63*] and metallic parts [*64*].

Nanofluid flow within a horizontal pipe was numerically investigated using CFD single-phase model simulations. The influence of CuO nanoparticles on water-based fluid has been highlighted in [52]. Three-dimensional CFD simulation results are obtained on thermal boundary layer analysis for CuO-engine oil laminar flow in a pipe [*69*] and the heat transfer enhancement for Al_2O_3-turbine oil nanofluid flow in a pipe [*70*].

For the impact of CuO-MWCNT nanoparticles on the heat transfer of oil nanofluid in a pipe [79]. The Al_2O_3-water nanofluid is considered with both constant and temperature-dependent thermophysical properties with volume fractions between 0.1% and 5%. Simulations were carried out in the developing and hydrodynamically developed regions. We compared the accuracy of results using temperature-dependent and constant thermophysical properties, the first one was found to be more accurate. Novel correlations were proposed for the hydrodynamic entry length and the thermal entry length using the temperature-dependent thermophysical properties. The results show that nanofluids require lower pumping power compared to the base fluid to provide the same level of heat transfer coefficient. We found that nanofluids with a volume fraction below 0.6% have a favourable PEI. Simulations of three-dimensional steady-state laminar flow within a horizontal pipe with CuO and multi-walled carbon nanotubes (MWCNT) and engine oil as a base fluid are also performed. The simulation for different nanoparticle volume fractions under a constant heat flux aimed to evaluate and compare the effect of different volume concentrations of CuO and MWCNT with different mixture ratios on convective heat transfer. A second-order discretization method is applied to solve equations and SIMPLE algorithm is used for pressure-velocity coupling applying CFD code. For the CuO-oil nanofluids' thermal boundary layer thickness, a formula was given depending on the volume fraction [Khalili].

2.5. Nanofluid fluid flows above surfaces

The steady/unsteady, laminar, incompressible/compressible MHD boundary layer flows driven by moving boundaries are among the classical problems of theoretical fluid mechanics (see the basic book by Schlichting). Our main aim was to study the heat and mass transfer and the influence of the parameters on the velocity and temperature field. The similarity transformation technique is adopted to find solutions to the Navier–Stokes equation reducing the system of nonlinear partial differential equations of the momentum and energy equations to a system of nonlinear ordinary differential equations. Solutions to the resulting equations were sought analytically or numerically for the given boundary conditions. Although for Navier-Stokes fluid flow problems it is very rarely expected that the solution is given in closed form, we have been able to produce this in several cases for highly nonlinear systems of differential equations obtained after the similarity transformation [10,40,41,*42*,49,50,56,59,*62*,67]. The solution has also been produced in the form of a power series [*78*]. In other cases, the solution of the system of equations was obtained using the HAM [66,76,80,81] or numerically using Maple/Matlab software with the Runge-Kutta-Fehlberg fourth-fifth order technique [20,28] or bvp4c technique [65,72,74,75]. CFD simulations are performed in [22,*25*]. Solutions for the velocity, temperature, skin friction, drag, heat transfer etc. were provided for different scenarios of the fluid flows. We analyzed several practical applications, considering Newtonian and non-Newtonian fluids, including many types of nanofluid flow on a stretching, porous sheet and curved surfaces with slip conditions, in a magnetic field and heat transfer [Saeed].

We considered *Newtonian fluid flows* above stretching/shrinking surfaces. The usual *stretching sheet problems* arise in polymer extrusion processes that involve the cooling of continuous strips extruded from a dye. In [67,68], we examined the behaviour of injection and radiation effects on the laminar boundary layer flow of an incompressible, viscous Newtonian fluid in a porous material across a continually shrinking sheet with heat and mass transfer in a stationary fluid [68]. The impact of radiation on the laminar flow of dusty ternary nanofluid over porous stretching/shrinking sheet with mass transpiration was investigated in [67].

The control of the boundary layer flow due to the stretching sheet can be enhanced by introducing *magnetohydrodynamic (MHD) effects*. This can be done by taking an electrically conducting fluid above the sheet and in a magnetic field perpendicular or inclined to the plane. The steady axisymmetric flow of fluid with graphene nanoparticles between two infinite stretching disks is considered in [59] with accelerated velocity and magnetic field. In [72], we analysed the effect of an inclined magnetic field on the flow of a fluid film near an unsteady moving surface by describing the time-varying stretching velocity. A liquid film is usually formed on a surface by a series of practical drop impacts or by the impact of a jet of liquid on a solid plate. Under these conditions, the plate remains stationary, but the interface of the liquid film starts to move at a certain speed.

The flow of a fluid through *a porous medium* has numerous applications in industries dealing with polymer extrusion processes, glass blowing, metallurgical processes, and geophysical areas. A variety of equations have been used to describe the flow of a fluid through a porous medium as it is one of the important key factors in maintaining the temperature in the medium. These equations are merely approximations to the appropriate balance laws. A variety of ideas have been suggested to model the flow of mixtures, and one such approach is that which follows from the seminal works of Darcy and Brinkman.

Nanofluids, which are engineered colloidal suspensions of nanoparticles in a base fluid, offer several advantages in various applications. One of the most significant advantages of nanofluids is their remarkably enhanced thermal conductivity compared to traditional heat transfer fluids. The benefits of nanofluids are promising candidates in several applications where efficient heat transfer and heat management are essential, such as cases investigated by us. We examined water [80,81], oil [65,75] and kerosene [75] based fluids [28] and mono [22,*25*,27,28,*64*], hybrid [65,75,76,80,81] and ternary nanofluids [67], [Saeed].

While each paper dealing with *non-Newtonian fluid* flows may focus on different specific aspects of fluid dynamics and heat transfer, they likely share a common interest in understanding the fundamental principles governing these phenomena and their applications in various fields of science and engineering. The papers [*42*,56,66,73,74] focus on mathematical modelling and analysis of *Casson fluid* flows over stretching sheets and on the influence of nanoparticle's volume fraction on the flow dynamics and heat transfer characteristics. [56] examines magnetohydrodynamics (MHD) Casson fluid flow over a perforated stretching or shrinking sheet and the combined effects of MHD, Casson fluid behaviour, and slip conditions on the flow and heat transfer characteristics near the sheet. [66] discusses the stratified Casson fluid flow containing microorganisms incorporating microbial effects and activation energy, in a stratified flow configuration over a stretching surface. [73] examines the influence of magnetic fields, Marangoni effects, nanoparticle suspensions, and radiation on the boundary layer flow and heat transfer characteristics and [74] explores the behaviour of Casson nanoliquids, which exhibit non-Newtonian characteristics, over surfaces experiencing time-varying stretching velocities.

The *Oldroyd-B fluid flow* was investigated in [20] to understand the influence of heat generation/absorption on magnetohydrodynamics (MHD) flow impinging on an inclined stretching sheet with radiation. The study explores the effects of various parameters such as heat generation/absorption, magnetic field strength, and radiation on the flow and heat transfer characteristics. The research contributes to the understanding of complex fluid dynamics and heat transfer phenomena in MHD fluids, which have applications in various engineering and industrial processes.

We investigated the incompressible and compressible heat-conducting *time-dependent* boundary layer by applying the two-dimensional self-similar Ansatz and the analytic solutions can be expressed with special functions [*39*, 40, 41] to the original PDE systems of incompressible and compressible boundary layers with heat conduction and reduce them to a coupled non-linear ODE system. For the incompressible case, the ODE system can be solved with quadrature giving analytic solutions for the velocity, pressure and temperature fields. Due, to our knowledge there are no self-similar solutions known and analyzed for any type of time-dependent boundary layer equations including heat conduction. In [Barna], five different time-dependent non-Newtonian boundary layer models in two dimensions are investigated, including external magnetic field effects: power law, Casson fluid, Oldroyd-B model, Walter fluid B model and the Williamson fluid. For the first two models, analytical results are given for the velocity and pressure distributions, which can be expressed by different types of hypergeometric functions.

The physical significance of the entropy generation for the mixed convection time-dependent flow of *Cross-hybrid nanoliquid* due to the stretched surface at a stagnation point was discoursed in [65] by applying the role of thermal radiation under convective conditions. For hybrid nanofluid, engine oil is used as a base liquid with copper (II) oxide CuO and titanium dioxide TiO_2 nanoparticles.

In [61], the flow of a *micropolar fluid* over a stretching or shrinking sheet is investigated under magnetohydrodynamic (MHD) conditions—the effect of the microrotation on the skin friction coefficient, the velocity, and the temperature. The numerical results reveal that the micropolar flow may accelerate or decelerate depending on the values of the mass transpiration and the permeability of the porous sheet. Further, it is observed that the increase in the microrotation increases the skin friction coefficient. [76] presented *micropolar hybrid nanofluid* flow comprising copper and alumina nanoparticles over a vertical flat sheet. The mixed convection and additional forces such as magnetic field, thermal radiation, heat source, and thermal slip condition are adopted in this analysis. The outcomes of this work show that the higher volume fractions of copper and alumina nanoparticles improved the hybrid nanofluid viscosity, which results in the augmenting variation in the velocity and thermal distribution.

To reduce the fuel costs of ships one important parameter that directly affects speed, and power requirements is the hull resistance. Using CFD we investigated how surface roughness affects the hull flow *resistance* of a ship. The research quantifies the variation of drag at different surface roughness levels using special wall functions. The surface roughness significantly enhances turbulence within the boundary layer [77], [Ali]. The study includes an investigation of the interaction between drag coefficient and vessel performance, to improve hydrodynamic efficiency.

We designed experiments on *drag-reducing surfaces* and their optimisation on surfaces with adjustable geometries. Using 3D-printed blade riblets of the same thickness and height, but printed at different distances, measurements were performed in a towing tank at four different speeds in the BME laboratory [Hla]. In the future, we want to compare our experimental results with CFD simulation results.

2.6. Ferrofluid flows

The thermo-magnetic behaviour makes ferrofluids useful in various practical applications in industrial engineering and technological applications. We observed that an increase in the stretching of the surface will decrease the velocity, temperature and heat transfer coefficient [2, 3]. With HDM, we have achieved two bifurcation solutions, and one of the two solutions agrees well with our previous results [5]. The increase in the magnetic effect increases the velocity and thermal boundary layer thickness and heat transfer coefficient [7]. The velocity, the temperature profiles, and the heat transfer coefficient depreciate with increasing the values of porosity [18]. An increase in the volume fraction slows down the flow and increases the temperature and the heat transfer on the wall [19].

Co-authors: The work was done together with several colleagues from abroad. We have more than one joint paper with foreign co-authors, excluding our foreign doctoral students: US Mahabaleshwar 16 (India), L. Mátyás 14 (Romania), M. Guedda 7 (France), A. Saeed 7 (Pakistan), A. Lone Showkat 5 (Saudi Arabia), J. Singh 3 (India), HF Öztop (Turkey), T. Maranna 3 (India), AB Vishalakshi 3 (India), F. Mabood 2 (Canada), EA Algehyne 2 (Saudi Arabia), A. Vencl 2 (Serbia), B. Stojanovic 2 (Serbia)

Project proposals submitted: A *Horizon Europe Pathfinder* project proposal was prepared with partners Cetin Civata Turkish bolt producer, Bay Zoltan Research Center, University of Miskolc, OX-IT, and AVL Austria for the investigation of functional multi-material bolts and joints in 2022. The task of the University of Miskolc is the tribological examination of the coated surfaces of the developed fasteners. The proposal was not supported.

A *Hungarian-Turkish TÉT project proposal* was also submitted by us on the examination of the torque testing and frictional measurement of bolts with Cetin Civata in July of 2022. (under review)

A *Hungarian-Serbian TÉT project proposal* was submitted with the University of Belgrad on the tribological investigations of composite materials in November 2023. (under review)

As the above report shows, many researchers have worked together successfully on this project. Nine articles listed under 1.2.2 have been prepared/submitted for publication, most of which are under review and four manuscripts are in preparation. We would like to continue these topics and submit a new project proposal in the future.

On behalf of all participants, I would like to thank the National Research, Development and Innovation Fund for supporting the project.

30. April 2024

Gabriella Bognár