Final report of the research project 'Many-body systems in and out of equilibrium'

During the 5-year period of the project we worked on four subprojects, among which 'Random quantum magnets', 'Clusters in disordered systems', and 'Effects of network heterogeneities on dynamics' generated significant synergies through the common concepts of disordered systems and the methods of investigation such as the strong-disorder renormalization technique. Transferring concepts and intuitions between random quantum systems and random classical stochastic systems, the latter field being in some cases more comprehensible, was proven to be mutually productive. Results have been published in 40 scientific papers (further 8 are under review), having a cumulative impact factor of 129 and nearly 100 independent citations so far. In what follows, we present the achievements of the project in the four subfields in detail.

Random quantum magnets

In a series of papers, we studied the effects of various types of inhomogeneities on the entanglement properties of quantum many-body systems. We studied the entanglement entropy of random partitions in critical fermionic systems, where the sites belong to the subsystem with a probability p. The leading contribution to the average entanglement entropy is found to scale proportionally to the volume, with an additive logarithmic correction term [1]. We studied the scaling of logarithmic negativity between adjacent subsystems in critical fermion chains with various (random, aperiodic, and quasiperiodic) modulations, numerically calculating its lower and upper bounds. Generally, a logarithmically dependence on the subsystem size was found [2]. We considered random-singlet phases of antiferromagnetic spin chains, and determined the sub-leading terms in the entanglement entropy by the strong-disorder renormalization group (SDRG) method [3]. We considered a sublattice-symmetric free-fermion model on a one-dimensional lattice with random, power-law decaying hopping amplitudes, and addressed the question how far an analogue of the random-singlet state is valid for this model. We found an overall logarithmic divergence of entanglement entropy with a non-universal prefactor [4]. We used entanglement witnesses based on the Hamiltonian to detect entanglement in the XY chain in thermal equilibrium and determined the temperature bound below which the state is detected as entangled. We also considered post-quench states, and demonstrated efficiency of energy-based witnesses in detecting entanglement in various circumstances [5]. We also studied entanglement witnesses related to the entanglement negativity in the presence of integrability breaking perturbations [6]. We derived the exact reduced density matrix for the electrons in an analytically solvable model, in which the electrons described as a Luttinger liquid are coupled to Einstein phonons. We obtained analytical expressions for the electron-phonon entanglement and the mutual information [7]. We also presented an approach that can be used for calculating the two-site reduced density matrix in strongly correlated electron systems such as the Hubbard model [8]. We investigated front dynamics in the Harper model after two different initial settings, either with a different number of particles or with a different temperature in the two halves of the system. At the critical point, we found a power-law time dependence of the entanglement entropy, while the mutual information increases logarithmically [9].

In a series of studies, we explored the ground-state properties and the phase diagram of different variants of quantum Ising chains in the presence of both transversal and longitudinal fields by using different numerical techniques (DMRG, combinatorial optimization, SDRG). For random, antiferromagnetic couplings, random transverse fields, and a homogeneous longitudinal field, we identified a reentrant ordered phase, which is the result of quantum fluctuations by means of an order-through-disorder phenomenon [10]. For the homogeneous, antiferromagnetic quantum Ising chain, we found a mixed-order transition: the entanglement entropy and the spin-spin correlation function signaled a divergent correlation length, while the bulk correlation function has a jump [11]. For random, ferromagnetic couplings and random transverse fields, the ordered phase is found to vanish when the longitudinal field is switched on [12]. Using combinatorial optimization techniques, we studied the critical properties of 2d and 3d Ising models with random, antiferromagnetic couplings and a homogeneous longitudinal field at zero temperature. The phase transition is found to be of mixed order in 2d, while in 3d, it is compatible with that of the random-field Ising model [13]. For the ferromagnetic q-state Potts chain in the presence of a transverse field and a q-periodic longitudinal field, the transition is found to be of mixed order [14]. We addressed the question whether correlations between low-energy excitations of the random transverse Ising chain in the Griffiths phase are irrelevant as predicted by the SDRG approximation. In the distribution of the excitation energy obtained by the free-fermion technique, we pointed out corrections differing from those of uncorrelated random variables [15].

Based on a relationship with random walks, we derived exact lower and upper bounds on the lowest energy gap of open transverse-field Ising chains, which are explicit in the parameters of the model. Applying the bounds to the case of coupling-field correlations, a model which is relevant for adiabatic quantum computing, we determined the dynamical exponent [16]. In transverse-field Ising models, disorder in the couplings gives rise to an unfavorable, slower-than-algebraic scaling of the density of defects produced when the system is driven through its quantum critical point. By applying Kibble-Zurek theory and numerical calculations, we demonstrated in the 1d model that the scaling of defect density with annealing time can be made algebraic by balancing the coupling disorder with suitably chosen inhomogeneous driving fields [17].

Clusters in disordered systems

Population boundary is a classic indicator of climatic response in ecology. We captured the effects of quenched heterogeneities on the ecological boundary with the disordered contact process in one- and two dimensions with a linear spatial trend in the local control parameter. We showed by the SDRG method that under a quasistatic change of the global environment, mimicking climate change, the front advances intermittently: long quiescent periods are interrupted by rare but long jumps [18]. We demonstrated by numerical simulations that the edge of the colonized region (the hull) is a fractal with a dimension 7/4. Its width and length change with the gradient according to universal scaling laws [19]. We studied the time dependence of the local persistence probability during a nonstationary time evolution in the disordered contact process in d=1, 2, and 3 dimensions by the SDRG technique and simulations. It is found to decay logarithmically with time [20]. We also studied the local persistence in disordered contact processes with long-range interactions, and pointed out that the persistence in the late-time limit is a discontinuous function of the control parameter [21]. Motivated by long-range dispersal in ecological systems, we formulated and applied a general SDRG framework to describe one-dimensional disordered contact processes with heavy-tailed (such as power law, stretched exponential, and log-normal) dispersal kernels, widely used in ecology. Our results reveal that the more slowly decaying dispersal kernels lead to faster-vanishing densities as the critical point is approached [22].

In photosynthetic organisms the energy of the illuminating light is absorbed by antenna complexes and transmitted by excitons to the reaction centers. We supported experimental findings on fluorescense yield by Monte Carlo simulations and by results from statistical physics based on random walk approximations of the excitation in the antenna [23]. During continuous illumination, the system undergoes a correlated percolation transition, the fractal properties of which were explored by extensive simulations [24].

We showed that highly correlated sites in complex systems can be inherently disconnected. This was illustrated on the example of the disordered contact process by applying numerical simulations and the SDRG method. We concluded that the critical dynamics was well captured by mostly one, highly correlated but spatially disconnected cluster [25]. We studied the geometrical properties of rare regions in the transverse Ising model with dilution or with random couplings. We defined the energy cluster as the spin clusters having the smallest excitation energy, and showed that these are not compact [26].

Effects of network heterogeneities on dynamics

We considered the Kuramoto model, a paradigmatic model of phase synchronization of interacting oscillators on sparse random networks such as the Erdős-Rényi graph. By large-scale, massively parallel numerical integration, we confirmed the compatibility with the mean-field universality class. However, the corrections to scaling are found to be non-monotonic and much stronger than those of the model with all-to-all coupling [27]. We studied the synchronization behavior of the Kuramoto model on a large, weighted human connectome network, both in the absence and presence of an additive Gaussian noise. We observed power-law-tailed synchronization times with non-universal exponents overlapping with the range of human brain experiments [28,29].

Homogeneous, stochastic threshold models are known to display mixed-order phase transitions. We provided numerical evidence, that even in case of high graph-dimensional, hierarchical, modular networks, a Griffiths phase in the k=2 threshold model is present below the mixed-order transition point [30]. We also provided numerical evidence that the Griffiths phase is robust against the presence of refractory states and randomized time-dependent thresholds [31].

We investigated suscetible-infected-recovered (SIR)-like models on hierarchical modular networks, embedded in 2d lattices with the addition of long-range links. We showed that if the topological dimension is finite, non-universal power-law growth of the density emerges, and the topological disorder alters the critical behavior [32]. By performing simulations and scaling analyses of the SIR model, we showed that diffusion for all agents constitutes a singular perturbation that induces a distinct critical behavior [33]. We have written an invited review, which discusses the critical dynamics of simple nonequilibrium models on large connectomes [34].

We investigated critical dynamics on a fruit-fly connectome, using the Kuramoto model. The synchronization transition is found to be mean-field-like, while the width of the transition region is found to be enhanced due to the modular structure [35]. We also investigated the synchronization transition of the Shinomoto-Kuramoto model on networks of the fruit-fly and human connectomes. We determined numerically the crackling-noise durations with and without thermal noise and found extended non-universal scaling tails [36].

Power-law distributed cascade failures are well known in power-grid systems. We attempted to describe this phenomenon using a threshold model based on a second-order Kuramoto equation. We showed that weak enough heterogeneity, coming solely from the random self-frequencies of nodes, does not give rise to power-law distributed cascades. On the other hand, a strong enough heterogeneity destroys the synchronization [37]. We showed by dynamical simulations on the European and United States high-voltage power grids that a synchronization transition occurs by increasing the coupling parameter with metastable states depending on the initial conditions, so that hysteresis loops appear [38]. In 3d, we provided numerical evidence for a hybrid phase transition [39]. We also analyzed outage data in power systems, collected from various public sources to calculate the outage energy and duration exponents by power-law fits. Power-spectral analyses of the outage event time series have shown traits of self-organized criticality [40]. Frustrated synchronization and chimera states are expected to occur in Kuramoto-like models if the spectral dimension of the underlying graph is low ($d<4$). We provided numerical evidence that this really happens for the high-voltage power grid of Europe and also for the fruit-fly connectome [41]. We studied the level of heterogeneity of large high-voltage power grids of Europe and North America. We analyzed the power capacities and loads of various energy sources from the databases and found heavy-tailed distributions with similar characteristics [42]. We identified sensitive graph elements of the weighted European power grids by different methods. Bypassing was found to improve synchronization the best, while the average cascade sizes were the lowest with bridge additions [43].
Based on finite-size scaling in a fiber-bundle model, we demonstrated that, outside the localized regime, the load-bearing capacity and damage tolerance on the macro scale, as well as the statistics of clusters of failed nodes on the micro scale obey scaling laws with non-universal exponents [44].

Ground states of classical and quantum systems of interacting particles

A new formula has been elaborated for the partition function of systems of interacting quantum particles, bosons or fermions. It is classical and non-stochastic in the sense that it makes no reference to operators and their spectral properties and is written in terms of sums and ordinary integrals. The formula may prove to be useful for rigorous or numerical work [45]. Based on this result, we showed that in an infinite system of interacting identical bosons there is off-diagonal long-range order if and only if a nonzero fraction of the particles form infinite permutation cycles. In particular, there is Bose-Einstein condensation if and only if the diverging cycle lengths increase at least as fast with the number N of particles, as $N^{2/d}$ in d dimensions [46]. We showed that, in $d\geq3$ dimensions at low temperatures or high densities, bosons interacting via pair potentials that are both positive and positive type, form permutation cycles whose length diverges proportionally to the number of particles. Based on previous results, this implies Bose-Einstein condensation [47,48].

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