

FINAL REPORT

on the project NKFI K 128422

Project title: Dynamical modelling and stabilization of single track vehicles
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Period: 01 October 2018 – 30 September 2023

The aim of the project was the theoretical and experimental investigation of single track vehicle dynamics. Namely, we focused on the stabilization of unstable motions of underactuated vehicles. In the project, several scientific results were obtained by the analysis of simplified mechanical models of these vehicles. Overall, 25 journal papers [1-25] (23 in WoS journals [1-23]), 19 conference papers (16 in international conference proceedings) and 19 conference abstracts with oral presentations (12 on international conferences) were published in the frame of the research project. Due to space constraints, we cannot highlight all of our results in this report. The achievements related to the fundamental analysis of contact dynamics, contact damping, non-smooth systems and tyre dynamics are not detailed here.

We primarily focus on the path following control and on the balancing problem of single track vehicles, since these tasks are more closely related to the main goals of the project.

1 Path following control of single track vehicle models

The dynamics of the in-plane single track vehicle model (so-called bicycle model) was investigated in our project using different levels of complexity. As a first approach, the nonlinear dynamics of the single track vehicle having rigid wheels was investigated considering positive and negative caster lengths at the steered front wheel. Thorough bifurcation diagrams were constructed and all the attractors and repellers of the state space were identified for both forward and reverse motions. It was shown that for reversing, an unstable limit cycle exists together with the stable low speed rectilinear motion. Namely, large enough perturbations on the steered wheel can lead to dangerous oscillations during reversing.

The path following control (lane-keeping control) of the vehicle was also in the focus of our study, see the left panel of Fig. 1. The impact of the time delay in the control loop was analysed for the rectilinear motion. State feedback control was applied in the single track model with relevant time delay, and stability charts were calculated in the parameter plane of the control gains. To eliminate the negative effects of the time delay, different predictions were applied. The so-called finite spectrum assignment (FSA) approach was also considered, and the effects of the parameter mismatches on the controller's performance were identified. It was shown that the FSA can significantly reduce the settling times of the lane-keeping controller.

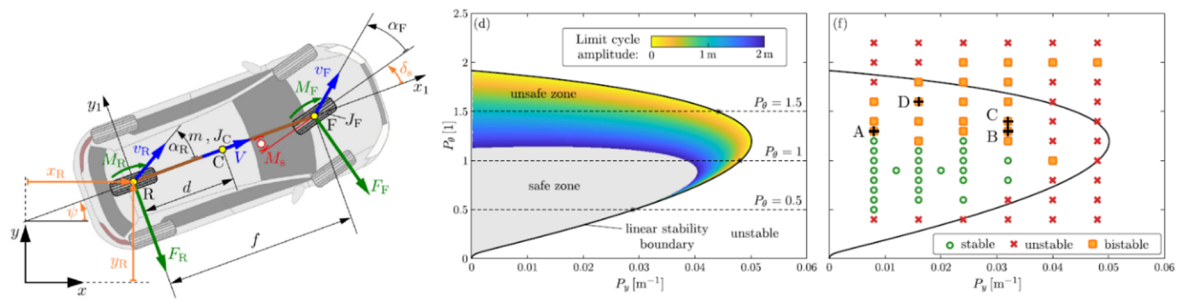


Fig. 1. The single track vehicle model (left), the theoretical nonlinear stability chart with the unsafe zone (middle), measurement results of the real vehicle tests: experimentally verified stability chart (right)

In order to validate the theoretical results, a unique experimental rig was designed for laboratory tests on small-scale vehicles, see Fig. 2. A digitally controlled conveyor belt was built, a special suspension with 5 degrees-of-freedom was manufactured for mounting of different types of vehicles (car, car-trailer combination, motorcycle), and a sensory system was attached to measure spatial position (lateral position, yaw and roll angles, trailer orientation) of the vehicles relative to the conveyor belt. The control of the autonomous vehicle was implemented by an NI cRio real-time controller, which enabled the precise tuning of the time delay with 1 ms resolution. This enabled us to emulate different realistic scenarios with respect to the digital effects and the time delay in the control loop.

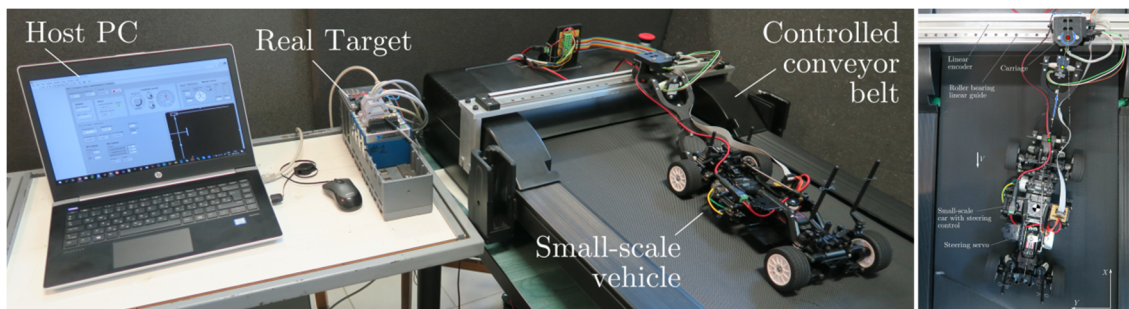


Fig. 2. The small-scale vehicle test rig constructed in the frame of the project. The real-time controller setup (left) and the vehicle with the special suspension system (right).

The linear stability chart of the lane keeping controller was accurately validated on the small-scale test rig. According to the experiments, the effects of the speed and the time delay clearly follow the theoretically predicted trends. The nonlinear vibrations in the stable domain of the control gains were also analysed and compared to numerical results. Namely, the domain of attraction of the linearly stable parameter ranges was calculated with the numerical bifurcation software DDE-Biftool, while the domain was also identified experimentally. The robustness of the lane keeping controller against perturbations was checked in this way, and unsafe zones were also charted, see the middle panel of Fig. 1.

Colleagues at the University of Michigan made experiments on the Mcity test track many years ago, where they tested the lane keeping controller of an autonomous car. In the different test runs, they varied the perturbations (initial conditions) and recorded the motion of the vehicle. These measurement data were now analysed by our research team. The theoretical domain of attraction was validated by the measurements, and a good agreement was established between the theoretical and practical results, see the right panel of Fig. 1. The conference paper submitted in this topic to the AVEC 2022 conference received the best paper award.

Autonomous driving is based on the fusion of different sensor signals, which often mixes the time delays related to the different sensors and data acquisition systems. In order to

investigate the effect of the sensor fusion, the time delays referring to the lateral position and the yaw angle were differentiated in the lane keeping controller. The stability of the lane keeping was analyzed with respect to various time delay combinations. It was shown that different time delays may lead to an enlarged stable control gain domain compared to the case when the delays are uniformly chosen. Moreover, it was explored that the increase of the time delay leads to a better performance in some cases, i.e., a faster decay of the oscillation can be achieved by adding extra delay to one of the signals. These results were published on the IFAC TDS 2022 conference, where the corresponding author of the paper received the Young Author Award.

The effect of an attached trailer on the stability properties of the lane keeping controller was also investigated by the group. Moreover, a controller for reverse motion of the car-trailer combination was developed. This control task is very similar to the balancing task of the motorcycle, i.e., both cases correspond to an under-actuated unstable system. For the reverse motion, theoretically calculated stable control parameters were validated on the experimental rig, which was improved to accomplish computer and/or human controlled reverse motion experiments. Computer controlled lane keeping and lane changing were carried out experimentally by the reversing car-trailer system.

2 Balancing of motorcycles/scooters

The control of autonomous vehicles is spanned over multiple levels or layers with varying complexity and time-scales. On the lowest level, typically actuators and motors are controlled directly, while higher levels of control are responsible for more complex operations, like path planning or obstacle avoidance. Using multi-scale simulation techniques, we have analysed how the different time-scales of control layers and time-delays between layers affect the overall robustness or responsiveness of a controller. Since it is possible to move certain control functionalities from the computer to the micro-controller (or vice versa), the optimal location for the implementation of a control task could be determined. The measurements and experiments carried out with an omni-directional platform provided important insights into the design of the self-balancing autonomous bicycle/motorcycle/scooter.

The researchers also extended previously applied models of digital control in several directions. In real control systems, both the input and the output are quantized. The thorough investigation of the case with input-only quantization revealed the similarities and differences between the phase-space structures of the input-only and output-only quantization cases. After that, the effects of twofold quantization were examined. It was shown that the interaction of the input and output quantization may lead to the collision of switching lines, which is a new global bifurcation phenomenon. Moreover, the so-called deadzone crisis – when the variation of the quantization parameter leads to a crisis event, turning the attractor to a chaotic repeller – was also explored in an inverted pendulum model.

Two mechanical models with different levels of complexity were used for the analysis of the motorcycle dynamics. A simplified model was constructed to investigate the balancing task at zero speed. In this model, an in-plane mechanism describes the yaw and steering dynamics, while an attached inverse pendulum imitates the lean of the motorcycle into the lateral direction. The complexity of this model enables the analytical linear stability analysis of the controlled vehicle. A more complex and accurate, spatial rigid-body bicycle/motorcycle/scooter model

(see the left panel in Fig. 3) was also implemented in a computer-algebraic software, and it was analysed numerically. The model was simplified by approximating the kinematic equation describing the effect of steering on the pitch motion of the motorcycle. Thanks to this modification, the governing equations of the spatial motorcycle model can be kept as a set of ordinary differential equations and the transcendent constraining equations can be decoupled. First, the applicability of a hierarchical controller was investigated for the balancing task without taking into account the time delay of the control loop. The useable control gain domain was determined considering both the higher level controller of the balancing task and the lower level controller of the steering servo. The results of the two different models were compared and the effects of the geometrical and inertial parameters of the motorcycle on the existence of the detected stable control gain domain were investigated.

In order to verify the theoretical results, the experimental rig built for the tests of small-scale vehicles was developed further. It was equipped by a bicycle/motorcycle model, see the left panel in Fig. 4. This small-scale model consists of commercial roller wheels and an RC servo actuator while the chassis and the steering mechanism was designed and manufactured by the group. The first experiments were accomplished, however, no stable control gain setup was detected in the theoretically predicted parameter domain. As the reason for the detected micro-chaotic vibrations, the low sampling frequency of the servo actuator was identified.

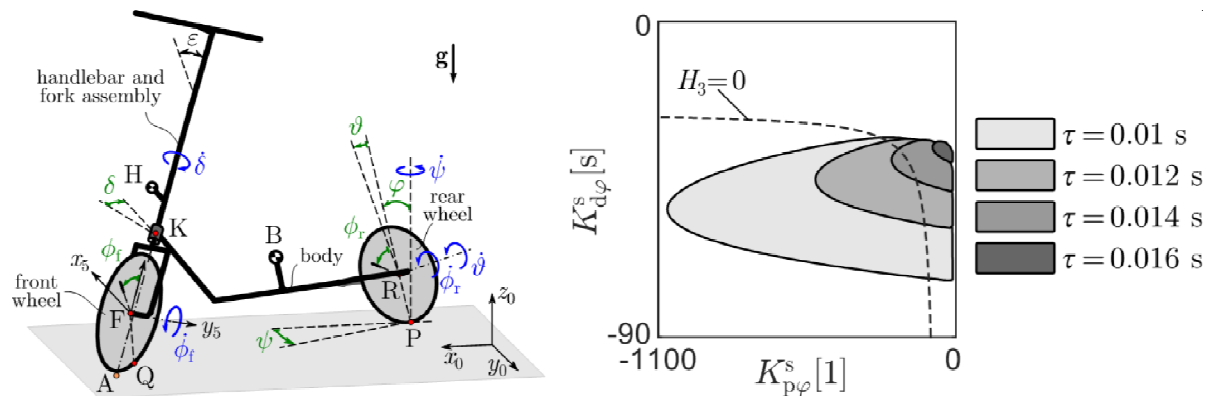


Fig. 3. The spatial mechanical model of the bicycle/motorcycle/scooter (left). The stability chart with the grayed stable control gain domains for different time delays of the control loop (right).

The mechanical models of the motorcycle were extended to capture the time delay in the steering control loop. The spatial mechanical model was compared to the simplified model. It was shown that the governing equations of the two different models coincide for some specific geometrical simplifications. Linear stability charts of the balancing motorbike were constructed on the parameter plane of the control gains of the higher (see the right panel of Fig. 3) and of the lower-level controllers. The critical time delay – above which the applied linear controller cannot stabilize the vertical position of the motorcycle – was determined numerically. Based on the mechanical models, it was explored that the balancing of the small-scale motorcycle of the experimental rig is possible with very small time delay only, and so the reconstruction of the test rig has become necessary. Simulations also showed that the accurate measurement of the roll angle of the vehicle relative to the vertical direction is a crucial task, namely, even a small uncertainty in this angle blocks the stabilization of the vertical position. Thus, the design of a new steering controller and an inertial sensor-based roll angle measurement unit has been started, see the middle panel of Fig. 4.

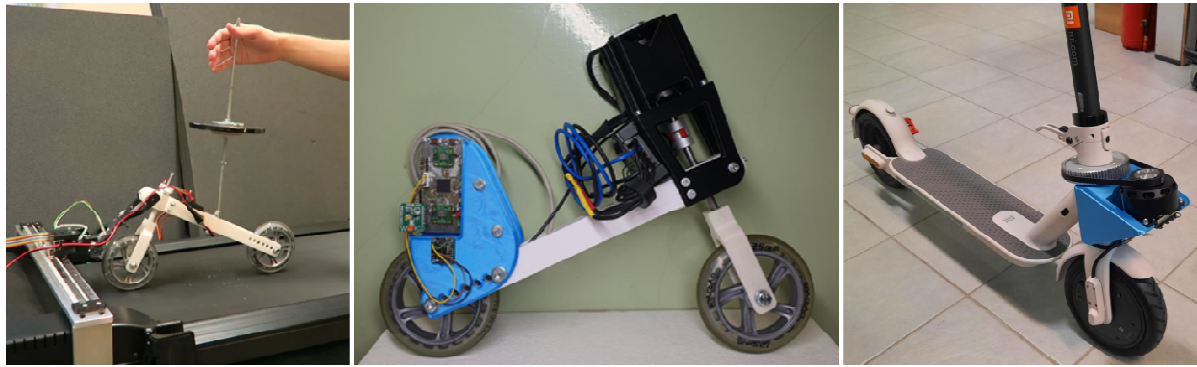


Fig. 4. The development of the experimental setup: the small-scale motorcycle on the conveyor belt (left), the small-scale motorcycle with improved steering servo and control unit (middle), a real e-scooter with an attached steering system (right).

The spatial motorcycle model was used to investigate the effect of the geometrical parameters (e.g. the wheel base, the trail, the rake angle) on the linear stability of the vertical position at zero longitudinal speed. Stability charts were constructed in the space of the control gains, and the most stable control gain setup – i.e., that leads to the smallest negative real parts in the roots of the characteristic equation – was determined. It was shown that while the rake angle has only a small effect on the size of the stable domain, the trail's effect is significant. Especially, a negative trail is beneficial; it enlarges the stable domain and it increases the maximal allowable time delay of the control loop. However, a negative trail causes instability when the motorcycle has non-zero longitudinal speed, i.e., it leads to a static loss of stability, or induces shimmy motion at higher speeds.

At this point of the project, we decided to focus on e-scooters, see the right panel of Fig. 4. For scooters, a simple solution exists by which positive trail can be changed to negative. Namely, turning the steering bar of the scooter by 180 degrees modifies the trail, and so negative values can be realized easily. This solution enables the normal use of the scooter for humans, while an enhanced automatic balancing task is possible by the designed controller. Moreover, locking the steering bar in the 90 degrees position may also enable the balancing by the front wheel drive (similarly like in the case of a Segway). Hence, two different controllers were designed by the research team: for the balancing with steering and for the balancing with front wheel drive. The construction of this specific e-scooter started in the last months of the project. The modification of the front wheel steering has already been finished, see the right panel of Fig. 4. The final experimental tests of the designed controllers and the validation of the theoretical results are ongoing tasks of the research team.

In summary, we can state that the research group achieved the goals set in the work plan almost completely, and even exceeded them in some areas. Based on the results achieved and with the help of the constructed experimental devices, many further publications are expected in the project's subject area.

Journal papers published in the framework of the project:

- [1] Gyebrošzki Gergely, Csernák Gábor: The Hybrid Micro-chaos Map: Digitally Controlled Inverted Pendulum with Dry Friction, *PERIODICA POLYTECHNICA-MECHANICAL ENGINEERING* 63: (2) pp. 148-155., 2019 *
- [2] Csernák Gábor: Analysis of pole acceleration in spatial motions by the generalization of pole changing velocity, *ACTA MECHANICA* 230: p. 2607., 2019

- [3] Gyebrószki Gergely, Csernák Gábor: Twofold quantization in digital control: deadzone crisis and switching line collision, *NONLINEAR DYNAMICS* 98: (2) pp. 1365-1378., 2019
- [4] Beregi Sándor, Takács Dénes, Stépán Gábor: Bifurcation analysis of wheel shimmy with non-smooth effects and time delay in the tyre-ground contact, *NONLINEAR DYNAMICS* 98 : 1 pp. 841-858. , 18 p., 2019
- [5] Várszegi Balázs, Takács Dénes, Orosz Gábor: On the nonlinear dynamics of automated vehicles - A nonholonomic approach, *EUROPEAN JOURNAL OF MECHANICS A-SOLIDS* 74 pp. 371-380. , 10 p., 2019
- [6] Beregi Sándor, Takács Dénes, Gyebrószki Gergely, Stépán Gábor: Theoretical and experimental study on the nonlinear dynamics of wheel-shimmy, *NONLINEAR DYNAMICS* 98, 2581-2593 (2019), 2019
- [7] Beregi Sándor, Takács Dénes, Gyebrószki Gergely, Stépán Gábor: Theoretical and experimental study on the nonlinear dynamics of wheel-shimmy, *NONLINEAR DYNAMICS* 4 pp. 2581-2593., 2019
- [8] Mi Tian, Stépán Gábor, Takács Dénes, Chen Nan: Vehicle Shimmy Modeling With Pacejka's Magic Formula and the Delayed Tire Model, *JOURNAL OF COMPUTATIONAL AND NONLINEAR DYNAMICS* 15 : 3 Paper: 031005, 2020
- [9] Vörös Illés, Várszegi Balázs, Takács Dénes: The effects of tire dynamics on the performance of finite spectrum assignment of vehicle motion control, *JOURNAL OF VIBRATION AND CONTROL* 28 : 1-2 pp. 45-57. , 13 p., 2022
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- [11] Csernák Gábor, Licskó Gábor: Asymmetric and chaotic responses of dry friction oscillators with different static and kinetic coefficients of friction, *MECCANICA* 56 pp. 2401-2414. , 14 p. (2021), 2021
- [12] Gyebrószki Gergely, Csernák Gábor, Milton John G., Insperger Tamás: The effects of sensory quantization and control torque saturation on human balance control, *CHAOS* 31 : 3 p. 033145 , 12 p. (2021), 2021
- [13] Beregi Sándor, Avedisov Sergei S, He Chaozhe R; Takács Dénes, Orosz Gábor: Connectivity-based delay-tolerant control of automated vehicles: theory and experiments, *IEEE Transactions on Intelligent Vehicles* 8(1) pp. 275-289, 2023
- [14] Horváth Hanna Zsófia, Takács Dénes: Stability and local bifurcation analyses of two-wheeled trailers considering the nonlinear coupling between lateral and vertical motions, *NONLINEAR DYNAMICS* 107 pp. 2115-2132. , 18 p., 2022
- [15] Lu Hangyu, Stépán Gábor, Lu Jianwei; Takács, Dénes: Dynamics of vehicle stability control subjected to feedback delay, *EUROPEAN JOURNAL OF MECHANICS A-SOLIDS* 96 Paper: 104678 , 13 p., 2022
- [16] Qin Wubing B, Zhang Yiming, Takács Dénes, Stépán Gábor, Orosz Gábor: Nonholonomic dynamics and control of road vehicles: moving toward automation, *NONLINEAR DYNAMICS* , 46 p., 2022
- [17] Vörös Illés, Takács Dénes: Lane-keeping control of automated vehicles with feedback delay: nonlinear analysis and laboratory experiments, *EUROPEAN JOURNAL OF MECHANICS A-SOLIDS* 93 Paper: 104509 , 13 p., 2022
- [18] Beregi Sándor: Nonlinear analysis of the delayed tyre model with control-based continuation, *NONLINEAR DYNAMICS* , 15 p., 2022
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- [20] Vörös Illés, Orosz Gábor, Takács Dénes: On the global dynamics of path-following control of automated passenger vehicles, *NONLINEAR DYNAMICS* 111 pp. 8235-8252., 2023
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- [22] Mihályi Levente, Takács Dénes: Linear Stability of Reversing a Car-trailer Combination, *PERIODICA POLYTECHNICA-MECHANICAL ENGINEERING* 67 : 2 pp. 87-93., 2023
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- [24] Vörös Illés, Turányi László, Várszegi Balázs, Takács Dénes: Small-scale Experimental Test Rig for Lateral Vehicle Control, *PERIODICA POLYTECHNICA-MECHANICAL ENGINEERING* 65 : 2 pp. 163-170. , 8 p. (2021), 2021
- [25] Dobák Dávid, Csernák Gábor: Negyed és fél járműmodell identifikációjának GP-LFM technikával való megvalósíthatósága, *MESTERSÉGES INTELLIGENCIA* 5 : 1 pp. 35-49. , 15 p., 2023