Final Report on Project KH 126853[1]

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At the beginning of the project Erdős, Mezei and Miklós published a preprint (see [1]) about a new class of bipartite degree sequences with a rapidly mixing switch Markov chain. (Along the project we decided to change the applied notion from swap to switch - in accordance with the general agreement of the literature.) Here the minimum and maximum degrees in the two color-classes are linearly bounded. Such a well behaving class of degree sequences will be mentioned in this report as a "good class". This was the first case where a class was known to be good for bipartite degree sequences, such that the analogue class for unrestricted degree sequences has not been known to be good. (Unrestricted means that there is no structural prescription - like being bipartite - on the realizations.) Erdős gave a lecture about these results at the 2018 ICGT conference in Lyon. He also gave lectures about these results at the universities of Christchurch, New Zealand; Brisbane and Sidney, Australia; and three further lectures in Illinois and Indiana, USA.

Later similar results were proved for degree classes where the maximum degrees are compared to the square root of the required number of edges. These latter results are analogous to the breakthrough results of Greenhill and Sfragara on unrestricted degree sequences. (Erdős, Mezei and Miklós, Soltész [2].)

The next logical goal has been to extend these new results for unrestricted degree sequences. As it turned out, the crucial difference between the proof methods of the bipartite vs. the unrestricted degree sequences lay in the circuit decomposition process. Our group, together with C. Greenhill and Lajos Soukup, spent the bigger part of the project to develop a unified approach to handle together all three main (unrestricted, bipartite and directed) degree sequence classes. We published a short report about these in [13].

The first real application of the unified approach was the study of the class of "P-stable" degree sequences. The notion of P-stability has a long and distinguished history: it was introduced by Jerrum and Sinclair in 1990 in connection with their study on evaluating the approximate values of 0-1 permanents. The class of P-stable degree sequences has been studied thoroughly ever since. We successfully proved that the classes of P-stable (unconstrained, bipartite or directed) degree sequences all have rapidly mixing switch Markov chains. The manuscript [14] was submitted.

It is well known that the class of P-stable degree sequences forms a natural boundary for the applicability of the Jerrum - Sinclair Markov chain. Our general approach seems to be restricted to the same region as the Jerrum-Sinclair method, but as it turns out, for our approach the P-stable boundary

[1] The notations [.] refer for the bibliography at the end of this report.

is a "soft" limit: in a somewhat surprising turn of events, we have found a "good" degree sequence class beyond P-stability (for which the switch Markov chain is rapidly mixing). This manuscript was also submitted, see [15].

Kundu's famous theorem assures that if D and D+1 are both graphic degree sequences, then there exist realizations which have perfect one-factors. (Not necessarily all realizations have this property.) Erdős, M. Ferrara and M. Hartke proved (in [17]) the space of all such Kundu realizations is connected under the switch operation. This result is the first necessary step to study the switch Markov chain of the space of Kundu realizations.

In the first year of the project Miklós has written the book "Computational Complexity of Counting and
Sampling" for Chapman and Hall/CRC (see [9]). Notable, the book contains several examples from the results of this project.

In the first year Erdős, Győri and Mezei, together with Colucci, finished their project on pairability problems on bipartite graphs (see [10]).

At the same time Győri, together with B. Ergemlidze and A. Methuku, almost finished a manuscript on characterizing directed graphs with every edge covered by a fixed number of cycles ([3]).

Győri, together with M.D. Plummer, Dong Ye and Xiaoya Zha, submitted another paper to Combinatorica ([5]). They have shown that in 3-connected, claw-free graphs there always exists a cycle passing through any given five vertices but avoiding any other one specified vertex. Furthermore, they proved that the result is sharp in an infinite family of such graphs.

Another paper of Győri ([4]), written together with Katona Gyula Y. and Papp László, is about the optimal pebbling number of the square grid. The problem of the pebbling problem has a long history and it is still actively studied topic. This paper applies a new technique for known problems.

Mezei, together with the other members of the group, participated the ICGT 2018 conference. Here he met I.
Hartarsky and started to work on the complexity of the 2-dimensional Critical Bootstrap Percolation
Models. In contrast with the upcoming result of Bollobás et. al. on undecidability in higher dimensions, they showed that the problem is NP-hard, but decidable. The corresponding manuscript ([7]) was submitted.

Chunqiu Fang, Győri and Jimeng Xiao studied the maximum number of colors in an edge-coloring of the complete graph with no properly colored copy of G. (see [20]).

In [22] Győri, N. Salia and O. Zamora generalized a result of Balister, Győri, Lehel and Schelp for hypergraphs. They determined the unique extremal structure of an n-vertex, r-uniform, connected, hypergraph with the maximum number of hyperedges, without a k-Berge-path.

In [19] Chunqiu Fang, Győri, Mei Lu, and Jimeng Xiao determined the exact value of the anti-Ramsey number for star forests and the approximate value of the anti-Ramsey number for linear forests.

In [21] Győri, N. Salia, C. Tompkins, and O. Zamora considered a version of the well-known Erdős and Sós problem of the Turán number of a tree for hypergraphs and multi-hypergraphs.

In [11] L. Colucci and Győri extended a result of Griggs and Yeh about the maximum possible value of the L(2, 1)-labeling number of a graph in terms of its maximum degree to oriented graphs.

In [12] L. Colucci, Győri, and A. Methuku improved on results of Hoppen, Kohayakawa and Lefmann about the maximum number of edge colorings without monochromatic copies of a star of a fixed size that a graph may admit.

In [6] Győri, N. Lemons, N. Salia, and O. Zamora studied the structure of r-uniform hypergraphs containing no Berge cycles of length at least k, for k at most r.

Soltész published the paper on "Even cycle creating paths" (see [26]). It is interesting, that using ideas from the Markov chain theory (which he got acquainted with through our project) the manuscript extends the previously known results widely.

Harcos Gergely and Soltész studied the problem, what is the maximal number of Hamiltonian paths in a complete graph, that the union of the edges of any pairs of the paths contains a (not necessarily induced) four-cycle. In [23] they improved the previously known bounds for this important special case.

In [24] M. Krenn, Xuemei Gu, and Soltész described some purely graph theoretical concepts to describe possible experiments to be used in quantum physics.

With the help of the grant, Erdős visited University of Delft in the March, 2018 where he, van Iersel and Jones finished a paper about non-reconstructable phylogenetic networks ([18]). In May, 2018 he visited University of Canterbury, Christchurch, New Zealand and University of New South Wells, Sydney. In the latter institution he started an active cooperation with Catherine Greenhill, what led to the papers [13] and [14]. In New Zealand Erdős, Semple and Steel described a new class of binary, rooted phylogenetic networks (the orchard networks). The published paper [16] already made a reasonable echo.

Finally, in October he visited the University of Illinois at Urbana Champaign and the University of Notre Dame, Indiana. At UIUC he was working with S. Kostochka and J. Balogh on some extremal graph theoretical problem. At the University of Notre Dame Erdős visited Z. Toroczkai and was studying a new model to generate scale-free degree sequences, a problem with possible practical applications. Here a new research paper is under the write-up process.

In October, 2018, with the help of the grant, Miklós participated at the conference RECOMB, Comparative Genomics in Quebec, Canada.

In January, 2019 Toroczkai visited our group in Budapest. In June, 2019 Erdős and Mezei reciprocated the visit at the University of Notre Dame. The described three visits culminated in the manuscript [25]. This describes a new kind of graph growing dynamics, which does not change the degree of any existing vertex. For example, this dynamics can provide an ever growing, fixed degree sequence of random regular graphs. The paper was submitted to a physics journal of very high impact factor.

In August, 2019 Mezei participated at the EUROCOMB 2019. Paper [13] is a relative short survey on our new results.

In October, 2019 Erdős visited van Iersel in Delft, The Netherlands and started to study the Nearest Neighbor Interchange (rNNI) moves on rooted phylogenetic networks. In October, 2019 Erdős and Mezei visited A. Francis and C. Greenhill in Sydney, Australia. They finalized the manuscript [14] with Greenhill. With Francis they proved that the tree-based, rooted, binary phylogenetic networks are closed under rNNI moves. The resulting manuscript is under preparation.