Final report on the NKFIH project

``Stationary processes in financial mathematics'' (KH 126505)

Rényi Institute, 1st December 2017--30th November 2019

The co-investigators of this grant have been Ngoc Huy Chau (NHC), Balázs Gerencsér (BG) and Miklós Rásonyi (MR, principal investigator). Our collaborator Laurence Carassus (Pole Universitaire Léonard de Vinci, Paris) also benefitted from the grant. The main achievements are grouped under five major topics below, followed by an overview of other activities.

1. Long-term optimal investment for non-Markovian prices

The textbook examples of non-Markovian processes with stationary increments are fractional Brownian motions (fBm) with Hurst parameters H in (0,1), where H=1/2 corresponds to ``ordinary'' Brownian motion (which is actually Markovian). The parameter H characterizes the memory of the given fBm and, at the same time, also the smoothness of its trajectories. Autocorrelation of the increments of an fBm is positive if H>1/2 and negative if H<1/2.

We were considering optimal long-term trading of an investor who seeks to maximize his expected profit facing liquidity constraints (which means that large trades are costly), in the setting of [1]. The price was assumed to follow a fBm.

Because of the lack of Markovian property, no Hamilton-Jacobi-Bellman equation can be set up for the value function of the problem and one needs to find some alternative, ad hoc approach.By casting the problem in an asymptotic setting (i.e. the growth rate of wealth on an infinite horizon is set as the objective function), we managed to find an asymptotically optimal trading strategy and we determined the optimal asymptotic growth rate. As far as we know, this is the first problem about utility maximization involving fBm for which a fairly explicit optimal solution has been found.

The conclusions can be stated in a rather simple form: when the extra cost of liquidity is a linear function of the trading speed (as argued by several papers in the literature) then the optimal expected return grows like T^{2H+1} as the time horizon T tends to infinity and this rate can be realized using a strategy that is, in a sense, Markovian (i.e. the current trading speed is a functional of the current price). When the extra cost of liquidity is a square-root function of the trading speed (as argued by another stream in the literature) then the optimal expected return grows like T^{3H+1}.

We also showed that the possible range of realizable growth rate is essentially covered by fBms: one cannot get a better or a worse rate trading in prices with stationary increments than the rates appearing in the fBm case. In other words, these processes cover the full spectrum of possibilities.

All the above results are written in [2]. The referees of the paper raised several interesting points which induced a painstaking revision, leading to substantial improvements. The result has been extended to a large class of Gaussian processes with stationary and positively correlated increments satisfying a certain correlation estimate, so the phenomena described above have a more or less universal character. It follows also that the growth rate is determined by the covariance structure (and not by some other, e.g. trajectorial properties).

In the follow-up paper [3] we are extending these results to the negatively correlated case, too. This paper is in preparation. In terms of generality, our papers' conclusions are going well beyond what was expected by providing general relationships between covariance structure and investors' achievable welfare.

We continued our efforts to understand fBm from an investment point of view. These led to [4] where the high-frequency region has been explored for investors with a mean-variance type objective. The limiting welfare has been explicitly determined as a function of the Hurst parameter H and the structure of optimal strategies has been analysed in detail.

## 2. Markov chains in random environments

The log-volatility of an asset price was modelled by a fBm in [5]. Recently, this has been shown to fit empirical data very well, see [6]. Our starting point was to regard this model as a Markov chain in a random environment (i.e. a Markov chain whose transition probabilities themselves form a stationary stochastic process).

In [7] we managed to prove convergence to an invariant distribution in weighted total variation norm and the law of large numbers in L^p for a large class of Markov chains in random environments. Our conditions are inspired by the standard assumptions of Markov chain theory (Lyapunov stability and minorization on small sets, see [8]). We combined these with a condition controlling the maximal process of the random environment.

This abstract result covers a broad selection of models, for instance, difference equations that can be regarded as discretizations of diffusion processes. In particular, our results apply to a discrete-time version of the fractional stochastic volatility model mentioned in the above paragraph. Results of [7] have been complemented in [9] where the Lyapunov stability condition has been considerably weakened while alternative minorization conditions have been introduced. In future work, we expect these results to serve as a basis for related large deviations estimates, too.

## 3. Stochastic optimal control

The standard method for proving the existence of an optimizer for utility maximization problems is passing through the (simpler) dual problem and then obtaining the primal optimizer from the dual solution. In [10] we managed to provide a method for tackling the primal problem directly, in the case where the utility function is defined on the whole real line.

In this way we may allow portfolio constraints of a very general type; we can handle a wider class of random endowments (e.g. when the investor has to deliver an option as well at the end of the trading period) than previous studies and we obtained pioneering results on the existence of optimizers for so-called large financial markets (see [11]). Further results about large financial markets have been obtained in [12] and [13] as well.

Our method found further applications in settings where model uncertainty is taken into account. In [14] optimal strategies are shown to exist in a model with transaction costs where the worst-case utility is considered over a family of possible models. This is the first continuous-time result of this kind in the literature. A discrete-time counterpart (without transaction costs) is presented in [15].

## 4. Informational arbitrage

How much is an investor willing to pay for learning some inside information that allows to achieve arbitrage? A particularly interesting case is where the inside information yields arbitrage opportunities in some sense. In [16] a general answer is provided to the above question in complete semimartingale models, by relying on an indifference valuation approach. To this effect, some new results are established on models with inside information and optimal investment-consumption problems are studied in the presence of initial information and arbitrage. It is also characterized when the value of informational arbitrage is universal in the sense that it does not depend on the preference structure.

## 5. Adaptive trading schemes for stationary prices

Stochastic gradient Langevin dynamics is a sampling method for distributions on high-dimensional spaces. It can be used to minimize possibly non-convex functionals on these spaces using noisy estimates/observations for their gradients. Hence it is widely used in machine learning applications. Our attention has been drawn to these methods for their potential use in algorithmic trading.

We have significantly extended the convergence analysis of such procedures. In [17] we treated the case of convex functionals in a setting where the gradient estimates contain dependent noise (satisfying an appropriate mixing condition). In [18] we went even further by relaxing convexity to a standard dissipativity condition. We proved the optimal convergence rate, ameliorating the known estimates (see [19]) in a spectacular way. We worked with the bounded Wasserstein distance while [19] used the quadratic Wasserstein distance. In [20] we investigated a similar procedure (called stochastic gradient Hamiltonian Monte Carlo) and obtained the optimal convergence rate for the quadratic Wasserstein metric.

Finally, in [21] the solvability of parameter-dependent Poisson equations has been analysed under a new set of conditions. These important tools of Markov chain theory play a prominent role in the analysis of recursive stochastic schemes. Future research will be directed to tackle such applications.

## Other activities

MR was awarded the degree ``doctor of the  Hungarian Academy of Sciences'' in December 2017 by the Doctoral Committee of the Academy.

We received the visit of Prof. Laurence Carassus (Pole Universitaire Léonard de Vinci, Paris) 4th-10th November 2018.

MR was co-investigator of the project ``Analysis of adaptive stochastic gradient and MCMC algorithms in continuous time'' that won 26000 pounds of support for the period July 2017--June 2018 from Alan Turing Institute, London. NHC was also involved in this research. NHC, BG and MR have also been supported by the ``Lendület'' grant 2015-6 since July 2015.

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[12] L. Carassus and M. Rásonyi. From small markets to big markets. To appear in Banach Center Publications, arXiv:1907.05593, 2019.

[13] L. Carassus and M. Rásonyi. Risk-neutral pricing for the APT. Submitted, arXiv:1904.11252, 2019.

[14] N. H. Chau and M. Rásonyi. Robust utility maximization in markets with transaction costs. Finance and Stochastics, 23:677--696, 2019.

[15] M. Rásonyi and A. M. Rodrigues. On utility maximization without passing by the dual problem. Submitted, 2019. arXiv:1801.06860

[16] N. H. Chau, A. Cosso and C. Fontana. The value of informational arbitrage. To appear in Finance and Stochastic, 2018. arXiv:1804.00442

[17] M. Barkhagen, N. H. Chau, É. Moulines, M. Rásonyi, S. Sabanis and Y. Zhang. On stochastic gradient Langevin dynamics with dependent data streams in the logconcave case. To appear in Bernoulli, 2019. arXiv:1812.02709

[18] H. N. Chau, E. Moulines, M. R\'asonyi, S. Sabanis and Y. Zhang. On stochastic gradient Langevin dynamics with dependent data streams: the fully non-convex case. Submitted, 2019. arXiv:1905.13142

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[20] H. N. Chau and M. Rásonyi.
Stochastic Gradient Hamiltonian Monte Carlo for Non-Convex Learning in the Big Data Regime. Submitted, 2019. arXiv:1903.10328

[21] A. Carè, B. Cs. Csáji, B. Gerencsér, L. Gerencsér and M. Rásonyi.
On the Poisson Equation of Parameter-Dependent Markov Chains. To appear in the Proceedings of the Conference on Control and Decision, Nice, France, 2019. arXiv:1906.09464

CONFERENCES AND TALKS

The most important talks and conferences are listed below.

Invited talk at the 2nd Conference on Mathematical Economics and Finance, Manchester, December 2017. (MR)

Talk at the Probability and Statistics Seminar of Rényi Institute, February 2018. (BG)

Contributed talk at the Workshop on Model Uncertainty and Robust Finance, Milan, March 2018. (NHC)

Invited talk at the Princeton-Rutgers Mathematical Finance Day, New Brunswick, April 2018. (MR)

Talk at the International Workshop on Differential Equations, CEU, Budapest, April 2018. (MR)

Invited talk the the Workshop on Analysis of Adaptive Stochastic gradient and MCMC algorithms,
Alan Turing Institute, London, June 2018. (MR)

Contributed talks at the 9th International Workshop on Applied Probability, ELTE, Budapest, June 2018. (NHC, MR)

Participation at the Workshop on Bayesian Computation for High-Dimensional Statistical Models, Singapore, August, 2018. (NHC)

Talk at the 9th Vietnam Mathematical Congress, Nha Trang, Vietnam, August 2018. (NHC)

Talk at the Workshop on Robust Techniques in Quantitative Finance, Oxford, September 2018. (NHC)

Talk at the Mini Workshop BayeScale on Markov Chain Monte Carlo and Stochastic Simulation, Ecole Polytechnique,
Palaiseau, France, September 2018. (NHC, MR)

Participation at the workshop ``Understanding the Diversity of Financial Risk - Machine Learning
Methods for Assessing Financial Risk'', ELTE, Budapest, October 2018. (NHC, MR)

Talk at the mathematics seminar of Princeton University, November 2018. (BG)

Talk at the Financial Mathematics Seminar, London School of Economics, November 2018. (MR)

Talk at the Young Researchers' Mini-symposium, R\'enyi Institute, November 2018. (NHC)

Lecture series at the Simons Semester on Stochastic Modeling and Control, Mathematical
Institute, Warsaw, 7th January--1st March 2019. (MR)

Talk at the Workshop on Recent problems of stochastic control theory, Warsaw, 28th January-- 2nd February, 2019. (MR)

Talk at the Conference on Stochastic modeling (in finance and insurance), Bedlewo, 11th--15th February, 2019. (MR)

Talk at the SIAM Conference on Financial Mathematics, Toronto, 4th--7th June, 2019. (MR)

Talk at 9th General AMaMeF Conference, Paris, France, 11th--14th June, 2019. (NHC)

Keynote talk at the Workshop on SDEs/SPDEs and their relationship to machine learning, Crete, 27th--30th June, 2019. (MR)

Talk at SIAM Conference on Control and Its Applications, June 2019. (BG)

Talk at the Random Structures and Algorithms Conference, ETH Zürich, Switzerland, July 2019. (BG)

Seminar at International University, Ho Chi Minh City, July 2019. (NHC)

Tutorial session at ISSAT International Conference on
Data Science in Business, Finance and Industry, Da Nang, July 2019. (NHC)

Talk at the 7th Asian Quantitative Finance Conference, Ha Noi, July 2019. (NHC)

Talk at ``Researchers' night'' in Renyi Institute, Budapest, September 2019. (BG)

Talk at the seminar of Department of Probability and Statistics, ELTE, Budapest, November 2019. (BG)

FULL LIST OF PUBLICATIONS

The list below contains all publications that have been supported by the grant KH126505. This fact is acknowledged in each of these papers.

[P1] N. H. Chau, Ch. Kumar, M. Rásonyi and S. Sabanis. On fixed gain recursive estimators with discontinuity in the parameters. ESAIM Probability and Statistics, 23:217--244, 2019.

[P2] N. H. Chau and M. Rásonyi.
Robust utility maximization in markets with transaction costs.
Finance and Stochastics, 23:677--696, 2019.

[P3] J. M. Hendrickx, B. Gerencser, B. Fidan.
Trajectory convergence from coordinate-wise decrease of quadratic energy functions, and applications to platoons. IEEE Control Systems Letters, 4:151--156, 2020. (alredy appeared)

[P4] P. Guasoni, Zs. Nika and M. Rásonyi. Trading fractional Brownian motion. SIAM J. Financial Mathematics, 10:769--789, 2019.

[P5] Zs. Nika and M. Rásonyi.
Log-optimal portfolios with memory effect. Applied Mathematical Finance, 25:557--585, 2018.

[P6] M. Rásonyi. On utility maximization without passing by the dual problem. Stochastics, 90:955--971, 2018.

[P7] N. H. Chau and M. Rásonyi.
Behavioural investors in conic market models.
To appear in Theory of Probability and its Applications, 2019. arXiv:1903.08156

[P8] N. H. Chau, A. Cosso and C. Fontana.
The value of informational arbitrage. To appear in Finance and Stochastics, 2019. arXiv:1804.00442

[P9] L. Carassus and M. Rásonyi.
From small markets to big markets.
To appear in Banach Center Publications, 2019. arXiv:1907.05593

[P10] M. Barkhagen, N. H. Chau, É. Moulines, M. Rásonyi, S. Sabanis and Y. Zhang.
On stochastic gradient Langevin dynamics with dependent data streams in the logconcave case.
To appear in Bernoulli, 2019. arXiv:1812.02709

[P11] A. Carè, B. Cs. Csáji, B. Gerencsér, L. Gerencsér, M. Rásonyi:
On the Poisson Equation of Parameter-Dependent Markov Chains, to appear in the Proceedings of the Conference
on Control and Decision, Nice, France, 2019. arXiv:1906.09464

[P12] B. Gerencsér and L. Gerencsér.
Tight bounds on the convergence rate of generalized ratio consensus algorithms.
Submitted, 2019. arXiv:1901.11374

[P13] L. Carassus and M. Rásonyi.
Risk-neutral pricing for the APT.
Submitted, 2019. arXiv:1904.11252

[P14] H. N. Chau, E. Moulines, M. Rásonyi, S. Sabanis and Y. Zhang.
On stochastic gradient Langevin dynamics with dependent data streams: the fully non-convex case.
Submitted, 2019. arXiv:1905.13142

[P15] H. N. Chau and M. Rásonyi.
Stochastic Gradient Hamiltonian Monte Carlo for Non-Convex Learning in the Big Data Regime.
Submitted, 2019. arXiv:1903.10328.

[P16] B. Gerencsér. Analysis of a non-reversible Markov chain speedup by a single edge. Submitted, 2019.
arXiv:1905.03223

[P17] B. Gerencsér and M. Rásonyi. On the ergodicity of certain Markov chains in random environments.
Submitted, 2019. arXiv:1807.03568

[P18] C. V. Kerckhove, B. Gerencsér, J. M. Hendrickx and V. D. Blondel.
Markov modeling of online inter-arrival times. Submitted, 2019. arXiv:1509.04857

[P19] A. Lovas and M. Rásonyi.
Markov chains in random environment with applications in queueing theory and machine learning.
Submitted, 2019. arXiv:1911.04377

[P20] M. Rásonyi and A. M. Rodrigues.
On utility maximisation under model uncertainty in discrete-time markets.
Submitted, 2019. arXiv:1801.06860

[P21] L. Nagy, M. Rásonyi.
Optimal investment in illiquid markets for prices with negative autocorrelation.
In preparation, 2019.

[P22] N. H. Chau and M. Rasonyi. Sensitivity analysis for ergodic maximization of asymptotic growth. In preparation, 2019.

[P23] N. Chada and N. H. Chau. On optimal annealing schedules for Langevin Algorithms. In preparation,
2019.