Project Closing Report

Following the research plan and its accepted revision, the project on Cooperation with Externalities – Theory and Applications was indeed organized into three main topics. I will now summarize the findings of each of these.

1. Cooperation with Externalities and Uncertainty

We analysed the extension of the classical cooperative game by introducing externalities and uncertainty at the same time in our published paper:

*Helga Habis & Dávid Csercsik, 2015. "**Cooperation with Externalities and Uncertainty**," Networks and Spatial Economics, Springer, vol. 15(1), pages 1-16, March.*

It is an important feature of the paper that it includes both important theoretical novelties; i.e. an extension of a game and its new solution concept, and the application of the proposed theory to a real case of electric power grid design.

We combine two directions of research, and allow for the presence of uncertainty and externalities at the same time by introducing the partition function form game with uncertainty (hereafter PFU-game). A PFU-game consists of two time periods, 0 and 1. In period 1 one out of a finite number of states of nature may materialize and conditional on the state, the players are involved in a particular partition function form game. An outcome therefore specifies a payoff-partition configuration conditional on each possible state of nature. A utility function is then used to assign a utility level to each profile of state-contingent payoffs.

We are interested in the appropriate definition of the core of a PFU-game. In this setting coalitions (forming some partition) are allowed to form in both time periods. Stability requires that a suggested allocation cannot be blocked by any coalition at any period, i.e. both before and after the resolution of uncertainty. We are interested in the case where agents cannot make fully binding agreements. Instead, agents will not stick to their agreements concerning the future if after the resolution of uncertainty, they are better off when deviating. Hence, we only allow for self-enforcing agreements in the spirit of Ray (1989) and Habis and Herings (2011). These considerations lead to the concept of the Sustainable Core. Extending the characterization of the Weak Sequential Core (Habis and Herings 2011); We say that an allocation belongs to the Sustainable Core only if – conditional on the state of nature – it belongs to the Recursive Core of the PFF-game related to that state, and moreover there is no coalition (forming some partition) in period 0 that can propose state-contingent Recursive Core elements of the game restricted to that coalition which gives its members higher utility.

An application of the PFU-game is the electrical energy transmission network, where consumers and generators need to find a stable allocation of power in- and outlets, taking into account possible line failures. Csercsik and Kóczy (2011) show that both negative and positive externalities exist in a static electrical energy transmission network game of generators and consumers, and therefore suggest the Recursive Core as a solution concept. Here we extend their example by the possibility of line failures in the grid, which creates the uncertainty. We show that the Sustainable Core can be used to find a solution to this extended game.

2. Assignment Games with Externalities

We have successfully published our findings on the comparison of the original assignment game and its extension in the following paper:

*Jens Gudmundsson & Helga Habis, 2017. "**Assignment games with externalities revisited**," Economic Theory Bulletin, Springer; Society for the Advancement of Economic Theory (SAET), vol. 5(2), pages 247-257, October.*

It is implicitly assumed in the vast majority of the literature that agents' preferences – or values in case of assignment games - are independent of how the other agents are matched. However, in many applications, for instance in case of labor markets, this assumption does not seem to be realistic. On the contrary, it is natural to assume that _rms competing in the same market care about which workers are hired by their rival firms. Thus, the profit of a firm ought to depend not only on its employee, but also on the way the rest of the agents are organized.

Analysing externalities in a matching framework is also important from a theoretical point of view, as it is well known that the core of a cooperative game might be empty in the presence of externalities. Thus, it is interesting to examine whether one can find a non-empty set of stable outcomes in this environment.

Externalities in matching problems began to receive attention following the wave of papers on cooperative games with externalities. The key aspect added through the externalities is that when a pair is considering whether to block a certain outcome, it has to take into account how the rest of the agents will react. These considerations are typically referred to as residual behavior in the cooperative game theory literature. Whereas these reactions do not play a role in problems without externalities, different assumptions and expectations about residual behavior lead to different outcomes being stable when externalities are present; ranging from the pessimistic approach of Aumann and Peleg (1960) to the optimistic one of Shenoy (1980).

We introduce externalities into assignment games by allowing the values of matched pairs to depend on how the rest of the agents are matched. We look for stable outcomes in the standard sense: an outcome is stable if it is individually rational and has no blocking pairs. However, it is not straightforward how one should define the notion of blocking in this environment. Here, we aim to cover as many behavioral assumptions or beliefs as possible by applying a very general definition of blocking. When agents are deciding whether to form a blocking pair, they take the values for all contingencies into account. According to their attitude towards risk or beliefs about the other agents, they calculate a threshold based on the possible outcomes and form a blocking pair whenever this threshold exceeds the sum of their current payoffs. By using this general definition, we avoid imposing any initial assumption on beliefs or residual behavior. In turn, we can distinguish different types of agents based on how they determine their threshold.

The first main result of the paper shows that a stable outcome in an assignment game with externalities always exists if all agents are pessimistic. Secondly, and of equal importance, we find that the slightest optimism could lead to non-existence of stable outcomes.

3. Self-enforcing Water Allocation Under Uncertainty

Growing competition over water resources is causing more and more disputes. We analyse the sharing a river problem applying cooperative game theoretic tools in situations where the river flow is uncertain, and agents are satiable. Given the stochastic nature of the problem, our focus is again on agreements that are self-enforcing.

We show that the set of stable and self-enforcing allocations is non-empty. In the case with no side-payments the only self-enforcing outcome is when all agents consume all their available water as long as they do not exceed their saturation point. Thus, there is no scope for real cooperation in this case. If we allow for side-payments than more options become stable, including the downstream incremental solution adopted to this setup.

The abstract of the paper was published in the SING 2015 Book of Abstracts. The full version will be published hopefully in 2020.