Final report

Renormalization group method in quantum theories

Introduction

In this project our goal was to treat the gravitational interactions in the framework of the functional renormalization group (RG in short) method. We intended to use closed time path (CTP) formalism, which could follow the evolution of expectation values instead of transition amplitudes. Furthermore this formalism could reveal certain shortcomings of the traditional RG method.
By using the CTP formalism for the RG method we realized, that traditional renormalization could not account for the contribution of the entangled states, and the nonlocal terms, appearing during the RG blocking. Additionally, there is an emerging instability (the nonboundness of the action), due to the nonlocality.
In order to reach our original goal, first we had to face with these deficiencies. The nonlocality plays an important role in these issues. The question can be approached by the gradient expansion, which can be considered as an infinitesimal nonlocality. In the project several results treated the problem of nonlocality by generalizing the wave function renormalization. The gravitational interaction was also approached by different aspects, e.g. in the framework generalized uncertianty principle (GUP), the quantum Einstein gravity (QEG), or its conformally reduced version.

Results

In this section we collect the results that were obtained in this project.
An important element of the project is the investigation of the gravitational interaction. Besides the RG method, the connection of quantum mechanics and gravity can also be analyzed in the framework generalized uncertianty principle (GUP), where the wavevector is maximized, which can be a low energy effect of the quantum gravity. We have reconsidered the 'particle in the box' problem in the framework of one-dimensional bandlimited quantum mechanics by using the infinitely deep trapezoid well potential with smeared out boundaries, and applying the nondegenerate stationary perturbation expansion to the determination of the low-energy stationary states by keeping track of the contributions of various orders to the direct and indirect GUP effect. It was shown that the relative energy shift of the low-energy stationary states of the nonrelativistic particle in the infinitely deep trapezoid-well potential show up a more significant contribution caused by the indirect GUP effect. We argued that perturbative treatment of the indirect GUP effect may not be possible for the potential well of finite depth [1].
According to the workplan we investigated such models where the momentum dependent wave function renormalization plays a crucial role. As one of the most important nontrivial model we investigated the 2-dimensional sine-Gordon (SG) model and their extensions by the functional RG method. We took care of the deep infrared (IR) and the far ultraviolet (UV) limits of the model. We found that the SG model is asymptotically safe; i.e., it possesses a non-gaussian fixed point (NGFP) in the UV limit. The UV NGFP is hidden by the singularity which nicely traces out the upper limitations of the model. The singularity has been identified in the IR limit, too. The similar behavior raises the question of the self-duality of the SG model for the UV and the IR limits [2]. We extended the SG model by taking into account the evolution of a further coupling in the momentum-dependent wavefunction renormalization. We showed that the obtained, extended SG model has no UV NGFP, and the UV

critical behavior exhibits an Ising-type phase transition. It reminds us of the massive SG model, where the IR limits showed a second order phase transition. This fact uncovered the dual relation between the extended SG and the massive SG models [2,E1].

In order to prepare the investigation of the scalar models in the presence of the heat bath we started to calculate the quantized one-dimensional anharmonic oscillator. We note that due to the strong truncation of the 1-dimensional model, we expect that the wave function renormalization plays crucial role. By using the functional RG method we numerically determined the energy gap for the model. The RG approach requires approximations which can introduce some regulator-dependence. We used the local potential approximation (LPA) and the Taylor expansion for the potential. We looked for the smallest deviation of the energy gap of the oscillator from its exact value as the function of the regulator parameters. The optimized regulator found in that manner is shown to be the generalization of the Litim's regulator and it provides an analytic evolution equation for the potential in $d = 1$. For the anharmonic oscillator with a double-well UV potential, this generalized Litim's regulator seems not to be the optimal one. Instead of the optimization via achieving the minimal sensitivity of the observable on the regulator parameters another optimization strategy can be followed. Then one looks for the regulator that enables one to reestablish the convexity of the numerically determined effective potential for the smallest value of the quartic coupling. We have found that the Litim's regulator appears in that case rather optimal instead of the generalized Litim's regulator introduced in the case of the single-well potential. It is argued that such a situation is due to the strong truncations in the gradient expansion and in the Taylor expansion of the potential [3].

We succeeded in taking a big step forward in the investigation of the RG method by using the closed time path (CTP) formalism. This work constitutes the most important part of the whole project. The CTP RG equation can take into account the contribution of the mixed states to the RG evolution of the models. According to the workplan we derived the Minkowksi version of the RG equations. We performed the calculations for both the blocked action (WH equation) and for the average effective action (Wetterich equation). As a further step we derived the CTP RG equation for the blocked action and applied it to the 3-dimensional O(1) model. We showed that the phase structure of the model can change significantly. Janos Polonyi gave a seminar talk in this topic [E4]. Imola Steib, who made her diploma thesis under my supervision, and later started her PhD studies in our department, also joined us in this work. We gave several talks in various forums about this project [E2,E3,E5,E6,E7].

During the derivation of the evolution equation in CTP formalism we realized that we need a bilocal potential term in the action in order to get interaction between the two time axes. Later it turned out, that the emerging nonlocality has much deeper reasons. A non-vanishing saddle point appears during the RG blocking, which contributes to the tree-level evolution of the bilocal potential. First we solved the tree-level RG equations. The modes that are eliminated during the RG blocking does not interact, therefore the independent mode approximation (IMA) is applicable. In the IMA the tree-level RG equations can be solved analytically. The results show that there are two phases in the model, as in the traditional treatment. The Wilson-Fisher fixed point cannot be seen at this level of calculation. The couplings belonging to the bilocal potential have CTP indices. The bilocal couplings with off-diagonal indices signal the entanglement between the modes of the physical system (IR modes), and the eliminated environmental modes (UV modes). This result is pioneering in the RG treatment, since the entanglement cannot be obtained by the traditional RG method. The entanglement phenomenon has no counterpart in classical physics, therefore we call our new RG approach as quantum renormalization group (QRG) method [4,E8,E9,E11,E13].

This part of the project turned my interest into the direction of investigating the entanglement in quantum informatics. I had possibility to join to a research which was led by Tamas Vertesi. In this

joint collaboration of the University of Debrecen and MTA Atomki, we investigated the amount of resources to simulate a bipartite hybrid quantum scenario. The scenario involves a trusted system and an untrusted part (a so-called black box system), which share a bipartite entangled quantum state. Local quantum measurements are carried out on the untrusted part, which influence the trusted part by remotely steering its quantum state. In [5] the quantum steering is simulated classically by substituting entanglement with classical communication between the untrusted and trusted systems. As a main result, we proved that infinite amount of classical communication is required from an untrusted party to a trusted party in order to simulate steering correlations arising from a pure two-qubit entangled state. The result demonstrated the power of quantum entanglement over the use of classical resources such as classical communication.
In the CTP formalism the tree-level renormalization has a key role, and it implies that we need a deeper understanding of the tree-level evolution. We expect, that the Einstein-Hilbert gravity with negative quadratic term (in its conformally reduced version) also needs tree-level renormalization. The simplified version of this model has been investigated first. We looked for the phase structure and the infrared behavior of the Euclidean 3-dimensional O(2) symmetric ghost scalar field model with higher-order derivative terms. We used the Wegner and Houghton's RG framework. We showed that there is a symmetric phase of the model in where no ghost condensation occurs. In the other phase a ghost condensate have been identified. Finiteness of the correlation length at the phase boundary hinted to a phase transition of first order [6]. We also considered the phase structure and the infrared behavior of the Euclidean 3-dimensional O(2) model including higher-derivatives in the kinetic term. We pointed out that higher-derivative coupling provides three phases and leads to a triple point in that RG scheme. The types of the phase transitions have also been identified [7].
In accordance with the workplan we investigated the Caldeira-Leggett (CL) model. The model contains an anharmonic oscillator linearly coupled to a heat bath. We treated the quantum-classical transition in the Caldeira-Leggett model in the framework of the functional renormalization group method. It is shown that a divergent quadratic term arises in the action due to the heat bath in the model. By removing the divergence with a frequency cutoff we considered the critical behavior of the model. We determined the critical exponents belonging to the susceptibility and the correlation length. We showed that they are independent of the frequency cutoff and the renormalization scheme [8].
We continued the research by investigating the bilocal action. The reason is twofold. On the one hand in the original workplan we intended to do wave function renormalization in the O(N) model. However the bilocal action can give a more general treatment than the wave function renormalization, since an infinitesimal distance in the bilocality can mimic the gradient expansion. Thus we can replace the wave function renormalization technique by considering the bilocal action. On the other hand the bilocal potential is an essential element in the QRG method, and the further plans should be performed only by considering bilocal potentials. The bilocal couplings become momentum dependent and this fact causes many technical difficulties. The bilocal action has not been treated either in the Euclidean single time action, therefore we started the investigation in this framework. The nontrivial saddle point gave tree-level contribution to the evolution. We also calculated the loop evolutions for both the local and the bilocal couplings. We obtained, that for the 3-dimensional ϕ^4 model the Wilson-Fisher fixed point disappears and a new fixed point emerges close to the separatrix belonging to the first order phase transition [11,E10,E12,E14,E15,E16,E17,E19]. We argued, that the nonlocality always appears in effective theories, and the RG blocking step inevitably introduces the nonlocal interactions.
We continued our investigation of the O(N) model with the Wetterich equation. We went far beyond the local potential approximation and we considered the generalization of the wave function renormalization including the quartic terms in the momentum. For simplicity we neglected its field dependence.

This study serves an important contribution to map out the phase structure of the O(N) model with more and more involved effective action. We note that this level of momentum dependence has not been treated yet in the literature We used the Litim regulator in order to have analytic beta functions. We calculated some critical exponents of the model in various dimensions. We obtained high precision values for the anomalous dimension and for the critical exponent belonging to the correlation length in the range of $2 < d < 4$ dimensions. The large N limit results have also been obtained correctly [9].
In order to discuss the occurrence of a periodic condensate in the Euclidean 3-dimensional ghost O(1) model, a modified version of the Wetterich equation. The RG flow equations are derived in the next-to-next-to-leading order of the gradient expansion. The characteristics of the Wilson-Fisher fixed point and the phase structure of the model have been determined numerically in the local potential approximation and in the next-to-leading order of the gradient expansion, when the periodic condensate has been modelled by a single cosine mode in one spatial direction. From the preliminary results important information is gained on further possibilities to improve the proposed RG scheme. We believe that our modified RG framework are worthy of future investigations [12].
We made a regulator dependent investigation in QEG. In order to have more reliable results than the ones available in the literature we took into account the term quadratic in the curvature in the action. The ultraviolet non-gaussian fixed point and its critical exponent for the correlation length were identified for different forms of regulators in case of dimension 3. We chose this dimension, since in d=4 there are further fixed points and singularity surfaces, which make the calculations extremely difficult. We searched for that optimized regulator where the physical quantities show the least regulator parameter dependence. It was shown that the Litim regulator satisfies this condition. The infrared fixed point has also been investigated, and it is found that the exponent of the correlation length is insensitive to the term quadratic in the curvature [10].
In our previous results it turned out that there is no such a non-perturbative regulator in Minkowski spacetime, which is Lorentz invariant. It implies, that we cannot derive the beta functions for the QEG model in Minkowski spacetime, because there is no Lorentz invariant regulator, therefore we cannot perform the loop integrals in d=4 dimensions in Lorentz invariant way. If we considered the model of QEG at the tree-level, we could not have problem with the regulators, because in this approximation every physical quantity is finite. However this treatment could run into another problem. As we showed last year, the closed time path (CTP) formalism inevitably introduces the nonlocality. It can be easily seen from the fact, that the real particles in Minkowski spacetime propagate, which is a nonlocal phenomenon, naturally. We should mention, that these problems were never raised in the literature, they were unexpected. Only those results, which were obtained in this project enabled us to realize and then to face with the problem of nonlocality.
These reasons forced us to simplify the question, which we raised in the beginning of the project. This is why we treated the simple scalar model, that come from the conformally reduced version of the QEG model. We derived the RG equations for the background field in Minkowski spacetime. In order to use the heat kernel expansion, we had to perform the frequency integral separately. As a result we obtained a cylindrically symmetric theory. We obtained, that there is a NGFP in the phase space. It is an UV attractive fixed point (a focal point), but the scaling exponents are slightly different from the ones obtained in Euclidean spacetime. This work is in preparation [14].
Due to the problems of the Lorentz invariance in the regulators, and the role of the nonlocality, we could not characterize the quantum-classical transition for the gravitational interaction at this level. However, after formulating in the conformally reduced QEG model in its more authentic Minkowski spacetime, we expect that our work will open up the possibility to investigate the pure QEG model with Lorentzian signature, too. Although the problem of the nonexisting Lorentz invariant regulator

remains unsolved, but in the framework of this simple model we can follow, whether the symmetry breaking is stronger or lower in the IR limit.

As we mentioned, the blocking transformation in the RG method generates nonlocal terms in the action. During the RG blocking step we integrate some degrees of freedom, and as a result we get an effective model. Every effective theory is nonlocal due to the regularization. In order to understand better the properties of the nonlocal models, we started to investigate a simple O(1) model in d=3. The nonlocality has been introduced via a bilocal potential. The results were very surprising. We obtained, that the bilocality gives a leading order tree-level contribution to the beta functions. It changes the phase structure fundamentally. We argued, that the bilocality is always generated in the RG method, and it contributes significantly to the RG flows. These results has not been known in the literature yet. It implies, that we should reinvestigate the models, since a leading order term is missing in the treatments.

We also obtained, that the evolution of the bilocal couplings make the model unstable. This result is general, the nonlocality usually leads to instability in the investigated models, because the action becomes nonbounded from below. It implies, that the effective theories are nonlocal, furthermore they are unstable. We note that every realistic model is effective. In quantum field theory the momentum integrals are infinite, therefore we need some regularizations. The regulators (e.g. a simple momentum cutoff) make some modes unobserved, therefore we obtain an effective model for the remaining observed modes. This is the case with the Standard model too, where without the UV cutoff, we have singularities due to the triviality problem of the Higgs sector and the Landau pole. It is also well-known, that new physics should emerge at high energy scales. We can conclude, that the Standard model is also unstable, however the observations are performed in short times, making the instability practically undetectable.

As a further step we performed the functional RG treatment of the 2-dimensional SG model in the framework of the WH equation. We improved the model by including the evolution of the bilocal term in the action. The RG blocking steps introduce a nontrivial saddle point, which gives a tree-level contribution to the RG evolution to the bilocal potential. We showed that the bilocal term can account for the Kosterlitz-Thouless type essential scaling of the correlation length ξ. This result could only be obtained so far in the framework of the Wetterich equation with smooth cutoff and taking into account the gradient expansion beyond the local potential approximation. The nonlocal interactions modify the phase structure of the models significantly. This has been demonstrated for the SG model. The phases, and the essential scalings obtained by gradient expansion could be recovered by the tree-level contribution coming from the bilocal potential. We also recovered the special role of $\beta^2 = 4\pi$, where the sine-Gordon model is equivalent to the Thirring model [13].

The formulation of the renormalization procedure in Minkowski spacetime arises further questions. The Minkowski propagator contains off-shell and on-shell contributions. The latter is diagonal in the momentum space, therefore cannot be local in the coordinate space. Furthermore, the formalism needs the introduction of complex couplings. Their imaginary parts start to evolve during the evolution, introducing new relevant couplings. In $d = 4$ the quartic coupling is irrelevant, making the model nonrenormalizable in nonperturbative sense. However the complex couplings can make the model renormalizable, which can be a possible solution to the triviality problem. We also obtained, that Minkowski-type model has richer phase structure [15,E18].

In order to formulate the loop improved QRG method, we should pair the Minkowski formalism with bilocality. We intend to work out this project in the near future.

Final remarks

In the project our main goal was to understand better the gravitational interactions in the framework of the RG method. There were several questions, which had to be answered before reaching our final goal. As we made progress in the project, several further issues emerged.
The most important result of the project was the QRG treatment of the scalar theories. We showed, that traditional RG method has a very serious shortcoming, namely, that they could not account for the contribution of the entangled states. Our results also showed, that the realistic models need nonlocal treatments, furthermore they exhibit instability. We think that these achievements in the field of RG technique are the most important results in the past two decades.
Summing up, we think, that the project was successful. The elements of the workplan has been successfully worked out. Although the final question could not be answered, but we solved several preliminary problems, and we made significant contribution to the RG method.
The project gave us a great help to achieve our goals. In its framework we wrote 12 articles in refereed international journals, which constitute the base of two PhD theses (József Kovács and Zoltán Péli). During the project Imola Steib started her PhD under my supervision. Now, she is writing her thesis in this research field with the title "Quantum Renormalization Group". The results helped me defending my thesis in habilitation. Furthermore, we successfully applied for the continuation of our project (No. KH126497)

References

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[14] S. Nagy, B. Fazekas, K. Sailer, I. Steib, Real time renormalization of conformally reduced quantum Einstein gravity, in preparation.

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Conferences, talks

[E1] International Seminar on Asymptotic Safety, 2015. Február 23, Debrecen (international web conference), giving a talk: Asymptotic safety in the sine–Gordon model.

[E2] Probing the Fundamental Nature of Spacetime with the Renormalization Group, Workshop, Nordita, Stockholm, Sweden, March 23-27, 2015, giving a talk: Real time renormalization.

[E3] Statistical Physics Day, April 10, 2105, giving a talk: Imola Steib: Renormálás valós időben (in Hungarian).

[E4] July 7-8, 2015, Debrecen, Department seminar, Janos Polonyi: giving a talk Dissipative forces in classical and quantum many-body systems.

[E5] 2015 ACHT Workshop, October 7-9, 2015, Leibnitz, Austria, Imola Steib: giving a talk: Real Time Renormalization.

[E6] 2015 ACHT Workshop, October 7-9, 2015, Leibnitz, Austria, giving a talk: Quantum renormalization group.

[E7] Local OTDK Conference, November 27, 2015, Debrecen, Hungary, Imola Steib: giving a talk: Kvantum renormálási csoport (in Hungarian).

[E8] [E1] QCD, Nonequilibrium Dynamics, Complex Systems, and Simulational Methods (Delta16 Meeting), 28-30 April 2016, Heidelberg, Germany, Sándor Nagy, talk: Quantum renormalization group.

[E9] [E2] Magyar Fizikus Vándorgyűlés, 24-27 August 2016, Szeged, Sándor Nagy, talk: Kvantum renormálási csoport (in Hungarian).

[E10] Magyar Fizikus Vándorgyűlés, 2016. augusztus 24-27., Szeged, Imola Steib, poster: Bilokális potenciál renormálása.

[E11] 8th International Conference on the Exact Renormalization Group (ERG2016), 19-23 Sep 2016, Miramare, Trieste, Italy, Sándor Nagy, talk: Quantum renormalization group.

[E12] 8th International Conference on the Exact Renormalization Group (ERG2016), 19-23 Sep 2016, Miramare, Trieste, Italy, ImolaSteib, poster: Renormalization of bilocal potentials.

[E13] 2015 ACHT Workshop, October 5-7 2016, Cakovec, Croatia, Sándor Nagy, talk: Quantum renormalization group.

[E14] 2015 ACHT Workshop, October 5-7 2016, Cakovec, Croatia, Imola Steib, talk: Renormalization of bilocal potentials.

[E15] FRG meeting, March 7-10 2017, Heidelberg, Germany, Imola Steib, talk: Renormalization of bilocal potentials.

[E16] ACHT Workshop, September 20-22 2017, Zalakaros, Sándor Nagy, talk: Inhomogeneous vacuum in bilocal theories.

[E17] ACHT Workshop, September 20-22 2017, Zalakaros, Imola Steib, talk: Renormalization of bilocal potentials.

[E18] 9th International Conference on the Exact Renormalization Group (ERG2018), 9-13 July 2018, Paris, France, Sándor Nagy, talk: Real time renormalization.

[E19] 9th International Conference on the Exact Renormalization Group (ERG2018), 9-13 July 2018, Paris, France, Imola Steib, poster: Renormalization of bilocal potentials.

Further talks

[E20] The 18th Debrecen–Katowice Winter Seminar, Hajdúszoboszló (Hungary), January 31 – February 4, 2018, Borbála Fazekas, talk: Double Walsh-Fourier series solutions of second order partial differential equations.

[E21] Numbers, Functions, Equations 2018, Hajdúszoboszló (Hungary), August 26 – September 1, 2018, Borbála Fazekas, talk: Walsh-Fourier series solutions of linear differential equations with error estimation.