Final report for the research project # **108947**

Optimization Methods for Cloud Computing and Communications

Summary: During the project we published 1 book, 5 book chapters and 119 research papers (85 in international research journals and 34 in the proceedings of international conferences)

Timeline: Our project started in September 1, 2013 and after a one year prolongation of the original four years program, it terminated at August 31, 2018. By the end of the fourth year we essentially obtained almost all the results we had planned in 2013; the aim of the prolongation was to finish publishing some of the most recent results and presenting them at some international conferences. Hence the period closing reports, submitted until September 2017, already contain almost every result of our group. In what follows we present the additional results only, which were obtained in the last 12 months.

The detailed description of the results of the first four years (a total of 20 pages excluding the references) is available at http://cs.bme.hu/kutatas/otka.html

The order of the presentation of the results follows that of the original research plan submitted in 2013.

Part 1

In 1966, Gallai raised the famous question whether in all finite connected graph there exists a vertex which lies on all longest paths. Though Walther in 1969 showed that the answer is negative, the topic has been extensively studied ever since. A family of extremal counterexamples to Gallai's question is called almost hypotraceable graphs, the paper of Wiener and Zamfirescu deals with the construction and structural questions concerning these graphs.

Wiener (Optimization Letters) proves that all claw-free graphs have a DFS-tree, such that the leaves different from the root have no common neighbour. This generalizes a theorem of Kano, Kyaw, Matsuda, Ozeki, Saito, and Yamashita and also implies a strengthening of a result of Ainouche, Broersma, and Veldman.

All known hypotraceable graphs are constructed using hypohamiltonian graphs. Wiener (J. Graph Th.) presents a construction that uses so-called almost hypohamiltonian graphs (nonhamiltonian graphs, whose vertex deleted subgraphs are hamiltonian with exactly one exception. This construction is an extension of a method of Thomassen. As an application, he constructs a planar hypotraceable graph of order 138, improving the best known bound of 154. He also proves a structural type theorem showing that hypotraceable graphs possessing some connectivity properties are all built using either Thomassen's or our method; and that no Grinbergian graph without a triangular region is maximal nonhamiltonian. Using the proof method he constructs a hypohamiltonian graph of order 36 with crossing number 1, improving the best known bound of 46.

Part 2

The paper of Mann (Software: Practice and Experience) investigates the impact of the choice of cloud simulator on the implementation and empirical evaluation of virtual machine placement algorithms. Specifically, it compares two cloud simulators (CloudSim and DISSECT-CF) regarding their support for the implementation and empirical evaluation of such algorithms.

The paper of Dräxler, Karl and Mann addresses the problem of automatically scaling, placing, and routing - in a joint optimization procedure - multiple network services sharing the same substrate network. Two algorithms are devised and compared, one based on mixed integer programming, the other a custom constructive heuristic.

Cloud computing is evolving into new directions, fog and edge computing being prominent examples. The aim of the paper of Mann (Optimization Problems in Fog and Edge Computing) is to give an overview about the wealth of optimization problems that arises in a fog or edge computing setting. The relevant metrics are discussed, as well as the role and place of optimization in the system lifecycle and the system architecture.

Part 3

Ervin Győri, Gyula Y Katona, László F Papp and Casey Tompkins investigated the optimal pebbling number of some induced subgraphs of the square grid graph. They determined this graph parameter when the subgraph is induced by at most seven positive diagonals.

For any positive integer d there is a graph whose optimal pebbling number is 2^d. Ervin Győri, Gyula Y Katona and László F Papp showed that the published proof of this result is wrong. They gave a new proof using dominating sets and proved the analogous statement for the optimal rubbling number.

Andrzej Czygrinow, Glenn Hurlbert, Gyula Y. Katona and László. F. Papp improved several estimates on the optimal pebbling number using the minimum degree of the graph. They proved that one of these new bounds is sharp.

The paper of Csongor Gy. Csehi, Adam Toth, Mark Farkas is part of the core algorithm of a complex software solution for truck itinerary construction for one of the largest public road transportation companies in the EU. The problem is to construct a cost optimal itinerary specifying the location and activity of the truck and the driver until the completition of the last routing task. The calculation of possible itineraries is a branch and bound algorithm. To make a sharp heuristic they run the same branch and bound algorithm (from each node) but with simplified data (hypothetical positions and simplified activities: no refueling, no road costs, etc.). They have reached significant gains in performance and quality compared to the previous approach.

Part 4

Applying game-theoretical tools for measuring the reliability of a network has become very common. Defining and analyzing attacker-defender games can give rise to new graph reliability metrics and in some cases it can shed a new light on some well-known ones. In his paper Szeszlér considers a set of games in which the attacker's aim is to hit a path chosen by the defender. He determines the values of these games and shows that the thus arising reliability metrics provide a generalization of weighted connectivity of graphs. He also proves that the values of the games and optimum mixed strategies for both players can be computed in strongly polynomial time.

Viamontes, Markov, and Hayes suggested the QuIDD data structure to simulate quantum computation, especially Grover's search. Katalin Friedl and László Kabódi suggest several possibilities to represent the quantum Fourier operator in this data structure. Although they are of exponential size but the fact that one can save a constant factor comparing to the case when the entire matrix is stored

still could be useful in simulations.

Katona Gyula Y, Soltész Dániel and Varga Kitti have proved that the minimum degree of a minimally-1-tough graph is at most n/3. For claw free graphs this bound is 2, since every minimally-1-tough graph claw-free graph is a cycle.

Since 1997 a considerable effort has been spent on the study of the mixing time of the **swap** (switch) Markov chains on graphic degree sequences. It is conjectured that the swap Markov chain is rapidly mixing on the realizations of every degree sequence, bipartite degree sequence or directed degree sequence. Péter L. Erdős, Tamás Róbert Mezei, István Miklós and Dániel Soltész widen the class of degree sequences for which rapid mixing is proven. As an application of these results, when a random bipartite or directed graph is generated under the Erdős—Rényi $G(n, p)$ model with mild assumptions on n and p then the degree sequence of the generated graph has, with high probability, a rapidly mixing swap Markov chain on its realizations.