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MŰSZAKI MECHANIKAI TANSZÉK

Development of mechanical models for the interlaminar fracture and vibration analysis of delaminated composite and sandwich plates (108414)

Principal Investigator:
András Szekrényes

1. Application of the first-, second- and general third-order plate theories to delaminated orthotropic composite plates.

The problem of asymmetrically delaminated plates was captured by using the second-order theory (SSDT) based on a novel approach. The plate was divided into two parts in the plane of the delamination and the kinematic continuity of the displacement field between the top and bottom parts was established through interface constraints. This method is called the method of two equivalent single layers. Using the basic equations of linear elasticity the equilibrium and governing differential equations of the plate were derived. A simply-supported plate with midplane delamination was investigated using the Lévy plate formulation in the code MAPLE. The continuity between the delaminated and undelaminated plate regions was established by using the equivalent stress resultants. The mechanical fields were calculated and compared to numerical (finite element) simulations. The fields are captured with high accuracy, however, the mode-II and mode-III J-integrals depend strongly on the position of the delamination along the thickness. It was shown that the second-order plate model can be applied only if the delamination is not too close to the free surface of the plate, otherwise the accuracy of the formulation decreases. The results in the form of an article were published in the journal MATHEMATICS AND MECHANICS OF SOLIDS (*Antiplane-inplane shear mode delamination between two second-order shear deformable composite plates*, appeared in 2017). The method of two equivalent single layers was also applied using the third-order plate theory by Reddy. It was shown that Reddy plate theory provides the very accurate description of the problem, however, if the delamination is very close to the free surface of the plate, then small errors are expected. The results appeared in INTERNATIONAL JOURNAL OF SOLIDS AND STRUCTURES (*Bending solution of third-order orthotropic Reddy plates with asymmetric interfacial crack*, 2014). The method of two equivalent single layers was also applied to general third-order plates (*Stress and fracture analysis in delaminated orthotropic composite plates using third-order shear deformation theory*, APPLIED MATHEMATICAL MODELLING, 2014).

Based on the interface constraints and the previous analyses asymmetrically delaminated plates were investigated using standard first-, second- and third-order shear deformable plates (FSDT, SSDT and TSDT). As a new aspect, the method of four equivalent single layers was developed. The method applies two equivalent single layers above and below the delamination plane. The so-called system of exact kinematic conditions was generalized to this case. Three novel conditions were formulated for second- and third-order plates: that is the continuity of the shear strains, its derivatives and curvatures. These conditions are important to control the oscillations of the displacement distribution along the thickness because of the perturbated fields in the vicinity of the delamination front. The advantage of the system of exact kinematic conditions is that it can be implemented to any plate theory for delamination modeling. Simply-supported delaminated composite plates were investigated using Lévy plate solution technique and it was shown that the first-order theory performs much better with four equivalent single layers than with two layers only. In the case of the second-order plate theory the displacement field is captured erroneously, in spite of that the results for the stress field and the distribution of the J-integral along the delamination front agree with the numerical calculations if the delamination is not very close to the free plate surface. The advantage of general third-order plate theory is that it provides more accurate description of the mechanical field distributions over the thickness of the plate. The governing equations of the model were derived and the state-space model was created for simply-supported plates. The analysis revealed that the third-order plate theory provides the accurate solution for the mechanical fields and the mode-II and mode-III energy release rates even in that case when the delamination takes place close to the free plate surface.

Based on the analysis of third-order plates and the solution of some relevant problems it was shown that the number of displacement parameters and stress resultants, for which the continuity conditions between the delaminated and undelaminated parts has to be imposed, is less than the number of available constants in the solution functions. It was revealed that in spite of the former the continuity of each field can be ensured by applying the theorem of autocontinuity. In accordance with the system of exact kinematic conditions the parameters of the displacement field are classified as primary and secondary parameters. The secondary parameters are expressed in terms of the primary ones. The continuity conditions have to be formulated only using the primary parameters, if the displacement field satisfies the system of exact kinematic conditions, then the continuity of the second- and third-order displacement function terms is satisfied automatically by ensuring the continuity of the so-called autocontinuity parameters. It was shown that the autocontinuity parameters are primary parameters too. A consequence of the theorem is that it is not required to impose the continuity of the membrane displacements between each neighboring layers: it is required to impose these conditions between two single layers above and below the delamination plane. The application of the theorem of autocontinuity and the consequence makes it possible to have continuous fields and exact solutions of the problem. The application of the updated system of exact kinematic conditions and the theoreom of autocontinuity was published in the journal APPLIED MATHEMATICAL MODELLING (*Semi-layerwise analysis of laminated plates with nonsingular delamination - The theorem of autocontinuity*) and appeared in 2016.

By using the method of four equivalent single layers it was shown that in the case of the second-order plate theory the large oscillations in the mechanical fields (in the through-thickness) direction of the plate lead to significant inaccuracies. Therefore, the system of exact kinematic conditions was complemented with the so-called shear strain control condition (SSCC). The SSCC means that in two or more points along the thickness of the plate the transverse shear strains are imposed to be the same. In accordance with Reddy's original theory the top and bottom surfaces of the plate are traction-free. However, this condition leads to the overstiffening of the system if the method of four ESLSs is applied. Instead of the dynamic condition, the SSCC condition can be applied to second, third- and higher-order plate theories. The governing equations and the state space models of some problems were created using the novel condition. By applying Lévy type boundary conditions the system of partial differential equations was reduced to system of ordinary differential equations. It was shown that the previously developed theorem of autocontinuity is valid even in this case and can be applied to define the continuity conditions between the delaminated and undelaminated parts of the plate. The results show that in the case of the second-order plate theory the SSCC eliminates the through-thickness oscillations in the mechanical fields. On the contrary, for the third-order plate theory, this condition induces oscillations again (similarly to the second-order theory without the SSCC), therefore it is a drawback in this case. During the calculations the displacement and stress components as well as the mode-II and mode-III J-integrals were determined. The comparison of the results of the second-, third-order plate theories and the 3D finite element calculations shows that the second-order plate theory with the SSCC condition provides accurate results in most of the cases. At the same time the so-called displacement multiplicator matrix and the invariant form of the equilibrium equations were derived. The article summarizing these results have been published in EUROPEAN JOURNAL OF MECHANICS/A – SOLIDS (*Nonsingular crack modelling in orthotropic plates by four equivalent single layers*) in 2016.

In the finite element model the distribution of the mode-II and mode-III energy release rates were determined by the virtual crack closure technique (VCCT). The disadvantage of the method is that it requires a 3D model with solid type elements. Based on the developed plate models the 4ESL method with SSDT (with the SSCC) and the TSDT could be the possible candidates to develop a plate finite element type for the delamination analysis of laminated composite plates.

The delamination modelling techniques using two and four equivalent layers together with the system of exact kinematic conditions were summarized in a DSc thesis written by the principal investigator. The thesis (entitled: *Nonsingular delamination modelling in orthotropic composite plates by semi-layerwise analysis*) was successfully defended in October, 2017 at the Hungarian Academy of Sciences.

2. Application of the first-, second- and general third-order plate theories to delaminated softcore sandwich plates.

The method of 4ESLs was also extended to the delamination analysis of softcore sandwich plates. A significant difference in sandwich plates compared to composite plates is that because of the very low moduli of the core the reference plane of delaminated regions coincide with the midplane of the facesheets. It was also shown that at the interface planes between the core and facesheets it is not reasonable to match the shear strains (as in the case of composite plates), because the shear locking (overstiffening) takes place. Because of the discontinuous shear strains the mathematical size of the delaminated sandwich plate problems is significantly higher than those of composite plates. The mechanical fields and J-integral were determined by the first, second and third-order shear deformable plate theories and comparison was made using FE results. The results showed that the SSDT and TSDT provide reasonable accuracy compared to 3D FE results, however the FSDT fails in modelling some scenarios. The models and conclusions were summarized in an article entitled "*Analytical solution of some delamination scenarios in thick structural sandwich plates*" which was accepted and is to appear in JOURNAL OF SANDWICH STRUCTURES AND MATERIALS in 2018. Also, the semi-layerwise models for composite and sandwich plates were presented in a conference in Bologna, in 2017 (*Semi-layerwise analysis of delaminated composite and sandwich plates*).

In the final stage of the project work we complemented the sandwich plate models with the effect of transverse stretching. The transverse stretching (normal deformation) was associated to the ESLs of the core material only, and the facesheets were treated as shear deformable layers. A simply supported delaminated sandwich plate subjected to uniform pressure was investigated using the FSDT and SSDT theories. The main result is that both theories give very accurate results for the J-integral. Moreover the transverse stretching has a significant improving effect in case of the FSDT. The improved sandwich plate models were documented in "*The role of transverse stretching in the delamination fracture of softcore sandwich plates*", submitted in 2018 to APPLIED MATHEMATICAL MODELLING.

3. Natural vibration induced parametric excitation in delaminated beams and plates

Based on the fracture mechanical analysis of delaminated plates the system of exact kinematic conditions was introduced in the vibration analysis of delaminated beams. The topic is the subject of researches since 1980. In fact each of the papers investigates the problem based on the same models: Euler-Bernoulli and Timoshenko beam theories. A special feature of the problem is that during the vibration the delaminated part shows significant delamination opening during the harmonic motion. In accordance with the literature the delamination opening is induced by the different inertia forces acting on the top and bottom beams of the delaminated part. In the course of this research it was shown that the problem is a coupled longitudinal-flexural vibration task. It was shown that the delaminated part is subjected to periodically changing normal forces as well. This means that the stiffness of the delaminated part is time-dependent and stability loss in the form of delamination buckling can take place during the vibration. It was shown that the distribution of the normal force along the beam length of the delaminated part is constant (in space) for the first and second natural vibration modes and changes exponentially in the case of third and fourth ones. These results were summarized in a journal paper appeared in JOURNAL OF SOUND AND VIBRATION (*Coupled flexural-longitudinal vibration of delaminated composite beams with local stability analysis*) in 2014.

Later, the stability of the system was examined by using Bolotin's harmonic balance method. Since this task can be solved only numerically the finite element discretization of the delaminated layered beam was carried out. The following matrices were derived for the delaminated beam: stiffness, mass and geometric stiffness. Also, a novel transition element was presented to capture the delamination tips. The free vibration analysis of the beam was performed to verify the analytical solution. The equation of motion for parametrically excited systems was modified to the special case, when the system becomes unstable only locally and the frequency of parametric excitation is equal to the free vibration frequency. The critical value of the internal normal force was determined by taking the finite determinant of infinite matrices. The results show that the dynamic stability limits are quite different than the corresponding static ones. Based on the stability analysis the so-called critical vibration amplitude – at which delamination buckling starts – was determined. The existence of the critical vibration amplitude was also proved experimentally. The mode shapes for the first four free vibration frequencies were determined based on the superposition of the free vibration mode shape and static buckling shape. The condition of constant arc-length was applied to obtain realistic delamination openings. The phase plane portraits for the midpoint of the top beam in the delaminated part was also determined and some special trajectories, like "icecream in cone", "pincers", "pac man", "light bulb", "carpet-beater" and "croissant" were discovered. The former results were published in EUROPEAN JOURNAL OF MECHANICS A/SOLIDS (*A special case of parametrically excited systems: free vibration of delaminated composite beams*) in 2015. The novel solution of vibration of delaminated beams was also presented in a conference in Prague (*Vibration and parametric instability analysis of delaminated composite beams, 2016*).

As a continuation the solution for delaminated beams was extended to plates and the problem was solved based on a linear approximation. It was shown that the delamination opening during the vibration of delaminated beams is the result of the action of periodic normal forces and time-dependent stiffness in the delaminated part of the beam. In the case of plates the same concept was applied, however the delaminated part in this case is subjected to normal forces and in-plane shear forces, as well. In an article (entitled: *Natural vibration-induced parametric excitation in delaminated Kirchhoff plates*) published in JOURNAL OF COMPOSITE MATERIALS in 2016, Lévy type composite plates were investigated and in the first step the frequency and mode shape calculation was carried out by using an analytical model. It was shown that the distribution of the normal and in-plane forces in the delaminated part of the plate is nonuniform in both directions. The local stability analysis of the delaminated parts was performed by using a finite element code developed in MAPLE. The distribution of the forces was approximated by piecewise constant functions along the length of the plate. The widthwise distribution of normal forces in Lévy plates is trigonometric (sine or cosine function), this aspect was considered by the following approximation. The static stability analysis was carried out in the traditional way (by assuming constant widthwise distribution of forces). The trigonometric widthwise distribution was considered using a harmonic balance approximation via second-order determinants (first kinematically possible solution). This way the ratio of the two solutions provided the so-called trigonometric load factor. In each case it was a constant factor with value of 1.44. Actually, it means that the maximum value of the trigonometric distribution should be 44% higher than the value of a constant distribution in order to reach the critical load and to induce buckling in the plate. This factor was utilized in the dynamic stability analysis with uniform widthwise distribution to calculate the corresponding critical loads with trigonometric widthwise distribution. The calculation was carried out for plates with various boundary conditions. An important finding of the work is that the critical amplitudes were determined. The subsequent appearance of the first and higher-order buckling eigenshapes were shown and it was concluded that the first eigenshape is the most dominant. The phase plane portraits of the midpoint in the delaminated area were created and some new trajectory shapes were discovered, like "glassware", "carpet beater", "carrot", "diving helmet", "nuts" and "strawberry" shapes.

4. Static stability (buckling) analysis of delaminated composite plates

Using the system of exact kinematic conditions and the method of two equivalent single layers (employing the classical or Kirchhoff theory) novel analytical and numerical (finite element) models were developed for the mixed buckling analysis of delaminated composite plates. The model was used to analyse the post-buckling state of delaminated plates under small deformation. The main idea of the model is that the distribution of the internal forces was determined in the delaminated part and it was shown that the distribution is nonuniform in both directions in the plane of delamination. To best of our knowledge, the mentioned model was the first considering this aspect of the problem. To consider the widthwise distribution of the force a local finite element model was developed and the local critical forces were determined by Bolotin's harmonic balance method using a second-order determinant. Based on the local force distributions the local critical forces and the mode shapes were determined. The mixed shapes were calculated by the subsequent summation of the forces in the function of the external axial load provided by a displacement control technique. The mixed buckling eigenshapes (i.e. when the plate buckles globally and at the same time the delamination buckles locally) were determined based on a linearized arc length criterion. The results of the model were presented in an article published in INTERNATIONAL JOURNAL OF SOLIDS AND STRUCTURES (*Progressive buckling of a simply supported delaminated orthotropic rectangular composite plate*) in 2015. Using the same concept the effect of delamination length on the critical value of the local normal force was investigated under different (Lévy-type) boundary conditions. The results were presented in an other article and was published in MATHEMATICAL PROBLEMS IN ENGINEERING (*Estimation of local delamination buckling in orthotropic composite plates using Kirchhoff plate finite elements*) in 2015. The same problem was solved by the first-order shear deformation plate theory (FSDT) and the results were compared to the Kirchhoff model, which showed that the effect of transverse shear is significant. The results were also presented in an international conference in Bologna (*Buckling analysis of a simply supported rectangular orthotropic composite plate with delamination*, 2015).

We also determined the buckled shape of delaminated composite plates caused by different type of loads with respect to the characteristic dimension of the delamination. The research involved the estimation of critical loads and the load carrying capacity in the post-buckled domain. For this reason orthotropic rectangular plates with through-the-width delamination are modeled using special types of Mindlin plate elements. The Mindlin plate finite element incorporates the first-order shear deformation theory. Different element types were developed for the delaminated and undelaminated regions as well as for the transition between them by using the system of exact kinematic conditions. The stiffness, mass and geometric stiffness matrices were determined. Some benchmark problems were solved to check the validity and the convergence of the developed FE type by comparing results to those obtained by the code ABAQUS. The numerical model is able to simulate laminated plates with through-the-width delamination.

The simulations were verified through experiments carried out on unidirectional laminates under uniaxial compression. The specimens contained a delamination with different size and location along the thickness. The edges parallel to the loading direction were traction-free. During the experiments the load-displacement response and the critical buckling force were determined. The very good agreement between the experimental and numerical results showed that the presented progressive finite element model is able to model the effect of delamination growth on the critical buckling load of uniaxial compressed plates. The novel part of the method is, that it also takes into account the local behaviour along the delaminated area, which is an important result because the plate can not only buckle globally, but also locally, which can significantly decrease the load carrying capacity. The relevant work entitled: *The effect of delamination on the critical buckling force of composite plates: experiment and simulation* was published in COMPOSITE STRUCTURES in 2017. Further conference presentations were held on this topic at *Mamek 2015* (Miskolc), *Tavaszi Szél 2015* (Budapest).

Some more result in this field is a geometrically nonlinear finite element model, which is able to simulate the post-buckling behaviour of the plate specimens. In this respect the von Kármán type nonlinearity was considered. The nonlinear solution is based on the Newton-Raphson iteration and was developed in the codes Mathematica/MAPLE.

5. Prediction of crack size in laminated composites by frequency measurement.

We also analysed the change in the eigenfrequencies because of the change of delamination length in laminated composite plates. We carried out an investigation based on the combination of modal analysis measurements with a novel 2D Finite Element (FE) plate model. The developed method is capable to determine the delamination growth based on the change in the measured frequencies due to crack propagation. The FE model is based on the classical laminated plate theory (CLPT) and it is capable to estimate the eigenfrequencies of a rectangular plate with through-the-width delamination and straight crack front under arbitrary boundary conditions. The FE model incorporated a 4-noded laminated Kirchhoff type plate element. The interpolation of the bending DOFs was done by the traditional Hermite polynomials, while the membrane displacements were interpolated by linear (Lagrange-type) functions. The stiffness, mass and geometric stiffness matrices of the delaminated and undelaminated regions were determined. A transition element between the delaminaed and undelaminated regions was also developed. Using this model the characteristics of the change of eigenfrequencies with respect to the delamination growth can be obtained. These results can serve as reference values, and according to the theoretical curves the actual size of the delamination can be estimated in any other plates based on the change in the measured frequencies. The advantage of the presented constrained FE model is its effective performance over the contact models. For the experiments plate specimens with through-the-width delamination were manufactured by a simple lay-up technology. For the tests a special rig was designed and manufactured providing simply-supported constraints along all edges of the plate subjected to uniaxial compression. The experimental results showed that the model can predict the effect of the delamination growth on the eigenfrequencies very well. Based on the good agreement between the experimentally and numerically determined frequencies further numerical examples were solved to demonstrate the influence of the geometry and lay-up on the frequencies of delaminated plates. The results show that – as expected – the frequencies decrease with the increase of delamination size.

By the combination of the measurement and calculation it is possible to develop a non-destructive failure detection technique, which can estimate the size of the delamination during the operation, so it is possible to change or repair the delaminated parts before leading to dramatic failure of the structures. The paper summarizing these results is entitled: „*Sensitivity analysis for frequency based prediction of crack size in composite plates with through-the-width delamination*" and is under appearance in INTERNATIONAL JOURNAL OF DAMAGE MECHANICS (2017).

The concepts and results of the buckling models of delaminated composite plates and shells, as well as the corresponding buckling and dynamic experimental results were summarized in a PhD thesis written by Zoltán Juhász. The first version of the document has been done and he plans to submit the thesis in the first half of 2018.

6. Dynamic stability analysis of delaminated beams and plates

In the final stage of the research project the effect of quasi-periodic excitation was investigated on the dynamic stability of delaminated composite beams. The quasi-periodic excitation was obtained by combining a longitudinal harmonic excitation with a random like transverse one, in which case the simplest approximation can be applied to obtain a solution. The mechanical model of the structure was created using the Euler-Bernoulli beam theory combined with longitudinal deformation. The basic idea of the model is the system of exact kinematic conditions. The beam was discretized using the finite element method and stability diagrams for the global structure were determined using the largest Lyapunov characteristic exponent. The amplitude and frequency of the time dependent longitudinal force and displacement excitation, the length of the delaminated part and the amplitude of the beam end corresponding to the first mode shape were examined on the dynamic stability regions of the constrained mode model. The results were compared with the case when there is no transverse excitation using Bolotin's harmonic balance method. It was found that in case of small beam end amplitudes the results of the two methods are in a very good agreement. The related paper entitled „*Quasi-periodic excitation in a delaminated composite beam*" has been published COMPOSITE STRUCTURES in 2017. The results were also presented in an international conference in Paris in 2017 (*Effect of additional inertia on the dynamic stability of delaminated composite beams*).

We continued to investigate further the dynamic stability of delaminated composite beams by incorporating experimentally measured and curve-fitted damping characteristics and higher-order beam finite element models. The relevant paper entitled „*Dynamic stability of a structurally damped delaminated beam using higher order theory*" has been submitted to MATHEMATICAL PROBLEMS IN ENGINEERING in January, 2018.

7. Differences compared to the original research plan

Finally we would like to justify the differences compared to the originally submitted research plan.

- In 2015 Zoltán Juhász joined the research project as a student.
- In 2016 Tamás Pölöskei joined the research project as a student.
- We did not investigated the results of the different experimental tests for mode-III interlaminar fracture. The reason for that was in the time of the research project this problem has been solved and published (Mehrabadi, F. A. (2014) APPLIED ADHESION SCIENCE, 2(1), 18.). Moreover parallel to the research project it was also shown by others that if the material is subjected to mode-III fracture then some microcracks at angle of 45$\circ$ to the delamination front take place and thus, the different tests lead to different critical energy release rates. (Johnston, A. L., Davidson, B. D. (2014) INTERNATIONAL JOURNAL OF SOLIDS AND STRUCTURES, 51(13), 2360-2369.). As a consequence, neither the MAPLE evaluation codes were created for the different specimens.
- The developed finite element types were not implemented into ANSYS. The reason was the time-consuming nature of the implementation. This problem is planned to solve in the future.

A lot of problems remained unsolved and we plan to continue the work in the near future. Finally, we are very grateful for the support of our researches during the past four years.

Budapest, 30th January, 2018.

Dr. Szekrényes András